On Spatial-Aliasing-Free Sound Field Reproduction using Finite Length Line Source Arrays

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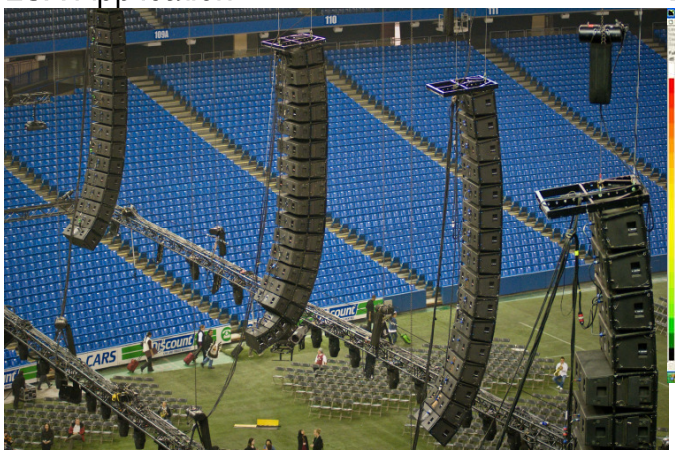
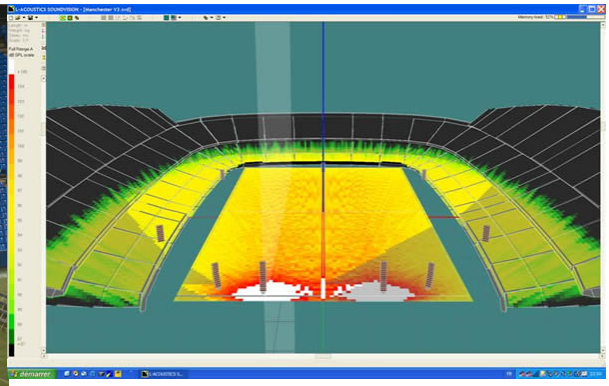
Introduction Line Source Arrays (LSA)

LSA Application



LSA Prediction



Wavefront Sculpture Technology (WST)
[Heil, 1992, 92nd AES Conv.]
[Urban, 2003, JAES **51**(10)]

recent approaches:
e.g. EAW Anya, Martin Audio MLA & OmniLine

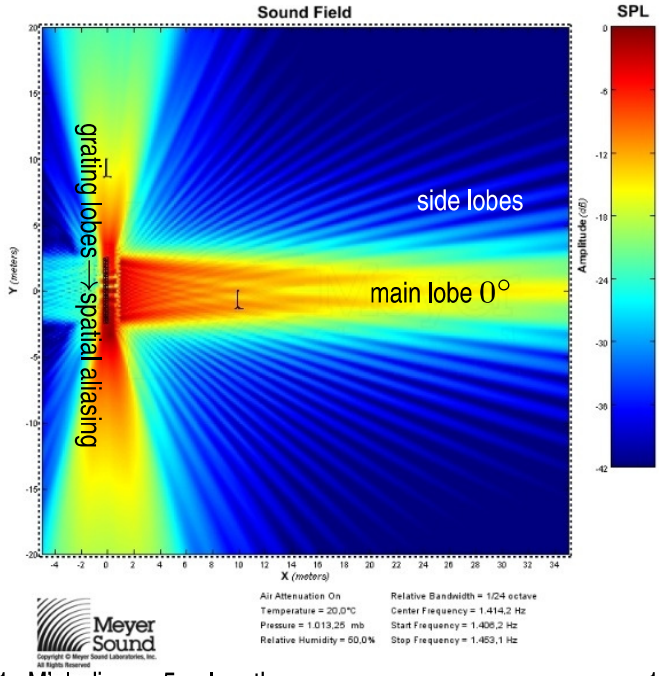
LSA Element

http://www.jblpro.com/press/jan09/JBL_IntlVerTec.jpg
http://www.turbosound.com/public/images/product_zoom/TFS-900H_front.jpg
http://www.ratsound.com/cblog/uploads/soundvision/MainBack.jpg

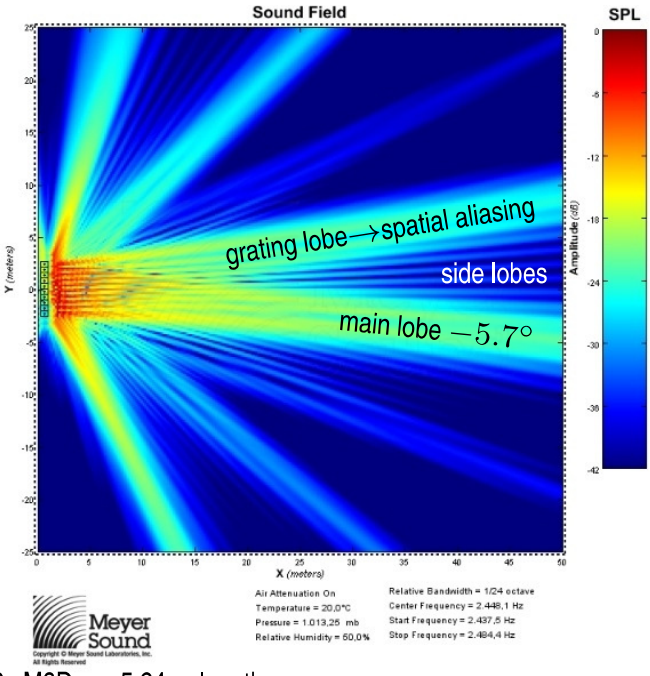
Version: 0.01 2014-10-06

Spatial Aliasing

Prediction with Meyer Sound MAPP Online Pro 4.4.0-9059[1]



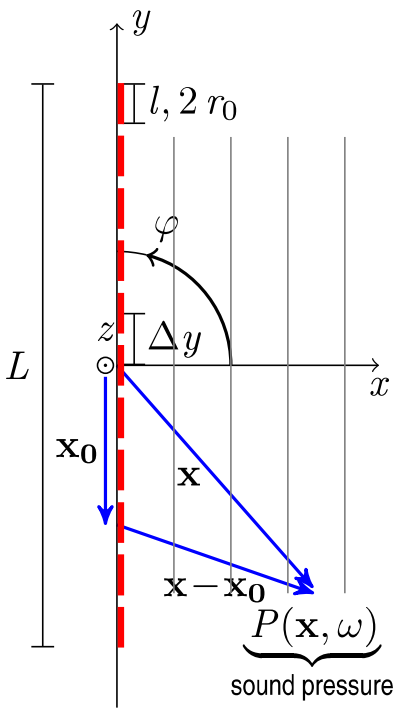
21x M'elodie $\longrightarrow$ 5 m length
no DSP, i.e. uniformly driven
f=1414 Hz, 1/24 oct.



10x M3D $\longrightarrow$ 5.24 m length
$\Delta\tau$=0.15 ms$\longrightarrow$steering φ=-5.7°
f=2448 Hz, 1/24 oct.

[1] Note that MAPP Online Pro is–in contrast to many other software–capable of revealing those artifacts. This does not imply that their products perform worse than others.

Problem Formulation

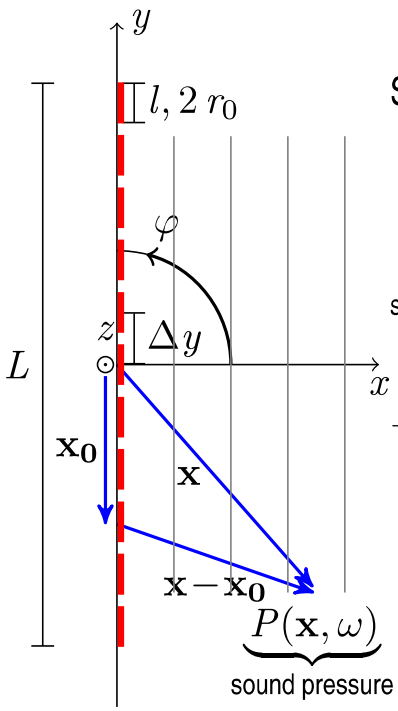


L...length of LSA in m
Δy...discretization step in m
N...number of pistons
l...line piston length in m
r_0...circular piston radius in m
φ...wave radiating angle
$\mathbf{x_0} = (0, y_0, 0)^T$...loudspeaker position on y-axis
$\mathbf{x} = (x, y > 0, 0)^T$...$xy$-plane
$P(\mathbf{x}, \omega)$...temporal Fourier spectrum of pressure

$$\underbrace{P(\mathbf{x},\omega)}_{\text{sound pressure}} = \sum_{n=1}^{N} \underbrace{D(\mathbf{x_0}[n],\omega)}_{\text{temporal filter}} \underbrace{H_n(\mathbf{x},\mathbf{x_0}[n],\omega)}_{\text{loudspeaker directivity}} \underbrace{\frac{\mathrm{e}^{-\mathrm{j}\,\frac{\omega}{c}\,|\mathbf{x}-\mathbf{x_0}[n]|}}{4\,\pi\,|\mathbf{x}-\mathbf{x_0}[n]|}}_{\text{spherical monopole}}$$

[Feistel, 2009, JAES **57**(6)]

Problem Formulation



Single Layer Potential / Rayleigh Integral

$$\underbrace{P(\mathbf{x},\omega)}_{\text{sound pressure}} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \underbrace{D(\mathbf{x_0},\omega)}_{\text{driving function}} \cdot \underbrace{\frac{\mathrm{e}^{-\mathrm{j}\,\frac{\omega}{c}\,|\mathbf{x}-\mathbf{x_0}|}}{4\,\pi\,|\mathbf{x}-\mathbf{x_0}|}}_{G(\mathbf{x},\mathbf{x_0},\omega)}\,\mathrm{d}z_0\,\mathrm{d}y_0$$

$\rightarrow$ interpretation with the spatio-temporal Fourier domain

$$\underbrace{P(\mathbf{x},\omega)}_{\text{sound pressure}} = \sum_{n=1}^{N} \underbrace{D(\mathbf{x_0}[n],\omega)}_{\text{temporal filter}}\,\underbrace{H_n(\mathbf{x},\mathbf{x_0}[n],\omega)}_{\text{loudspeaker directivity}}\,\underbrace{\frac{\mathrm{e}^{-\mathrm{j}\,\frac{\omega}{c}\,|\mathbf{x}-\mathbf{x_0}[n]|}}{4\,\pi\,|\mathbf{x}-\mathbf{x_0}[n]|}}_{\text{spherical monopole}}$$

WFS vs. WST

Wave Front Synthesis / Wave Field Synthesis (WFS)

Berkhout, A.J.; Vogel, P.; de Vries, D. (1992): "Use of wave field synthesis for natural reinforced sound." In: *Proc. of the 92nd Audio Eng. Soc. Convention, Vienna*, #3299.

$\rightarrow$synthesis of a virtual source with linear/planar loudspeaker arrays

Origin of **Wavefront Sculpture Technology (WST)**

Heil, C.; Urban, M. (1992): "Sound fields radiated by multiple sound sources arrays." In: *Proc. of 92nd Audio Eng. Soc. Convention, Vienna*, #3269.

$\rightarrow$synthesis of an almost spatial-aliasing-free sound field with uniformly driven line arrays for sound reinforcement

WFS vs. WST $\rightarrow$ strongly related ideas surfaced at the same time, different treatment at first sight, however same acoustic signal processing framework

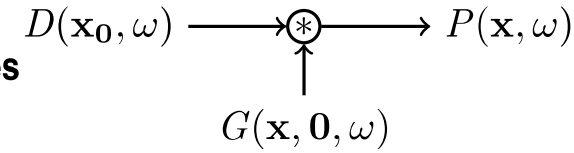
WST criteria [Heil1992]:

WST #1: ARF ≥ 0.82 $\rightarrow$ line piston (waveguide) array:

grating lobes attenuation larger than 13.5 dB in farfield

WST #2: $\Delta y < \frac{\lambda}{2}$ $\rightarrow$ no grating lobes, i.e. baseband signal's sampling theorem

Signal Processing Model for Sound Field Synthesis



- **Continuous** problem formulation with **monopoles**

$$\underbrace{P(\mathbf{x},\omega)}_{\text{sound pressure}} = \int_{-\infty}^{+\infty} \underbrace{D(\mathbf{x_0},\omega)}_{\text{driving function}} \cdot \underbrace{G(\mathbf{x},\mathbf{x_0},\omega)}_{\text{spherical monopole}} \,\mathrm{d}y_0$$

- Interpretation as a **spatial convolution** along y

$$P(\mathbf{x},\omega) = D(\mathbf{x},\omega) *_y G(\mathbf{x},\mathbf{0},\omega)$$

- This corresponds to a **multiplication** in the spatio-temporal spectrum domain

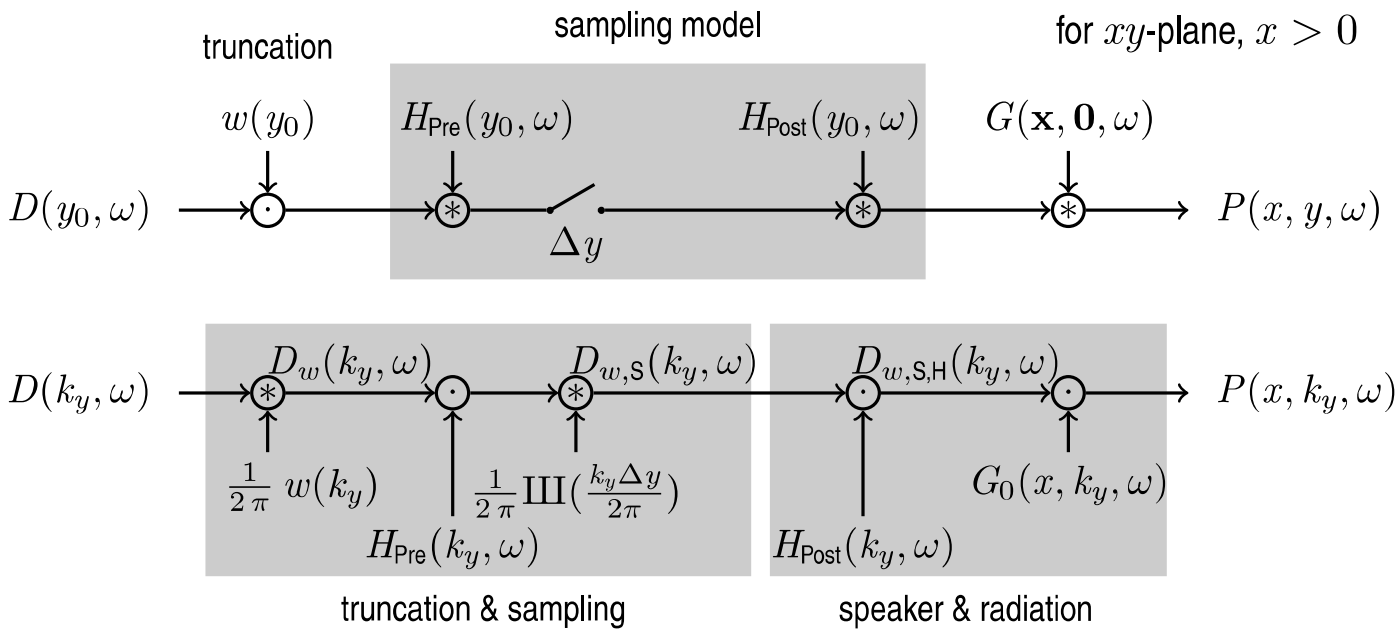
$$P(x,k_y,z,\omega) = D(x,k_y,z,\omega) \cdot G_0(x,k_y,z,\omega)$$

using the one-dimensional spatial Fourier transform along y

$$P(x,k_y,z,\omega) = \int_{-\infty}^{+\infty} P(x,y,z,\omega)\,\mathrm{e}^{+\mathrm{j}\,k_y\,y}\mathrm{d}y$$

[Ahrens, 2010, IEEE Audio **18**(8)]

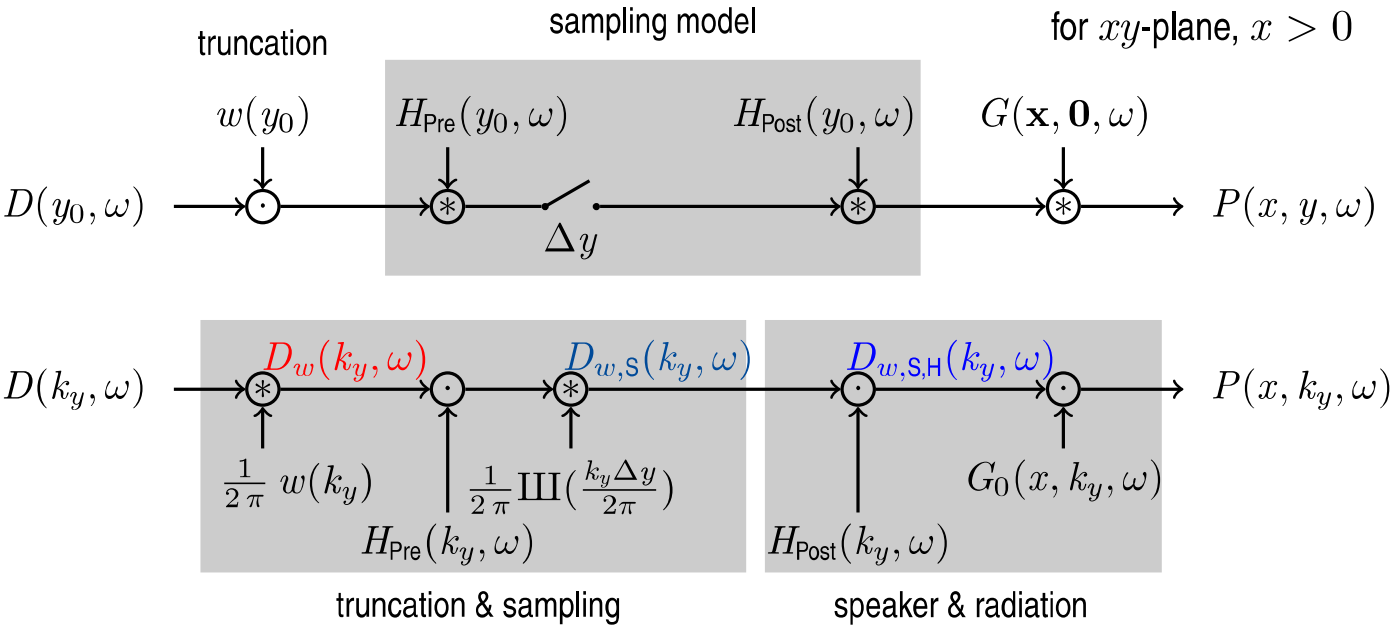
Signal Processing Model for Sound Field Synthesis



[Start, 1995, 98th AES Conv.]

[Start, 1997, PhD]

Signal Processing Model for Sound Field Synthesis



Farfield Directivities:

$D_w(k_y,\omega)$ ...continuous, finite length LSA

$D_{w,\text{S}}(k_y,\omega)$ ...discretized, finite length, monopoles LSA

$D_{w,\text{S,H}}(k_y,\omega)$...discretized, finite length, pistons LSA (product theorem, WST#1 derivation)

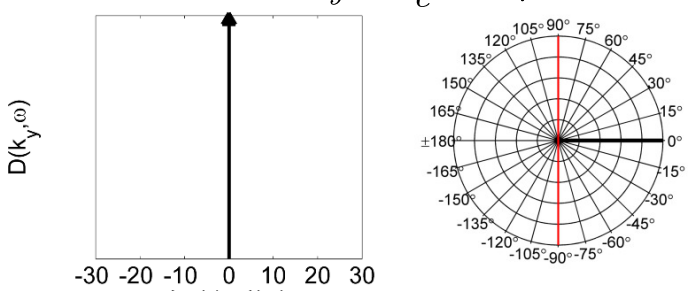
Driving Function

$$k_y = \frac{\omega}{c}\,\sin\varphi$$

uniformly driven, infinite, continuous LSA:

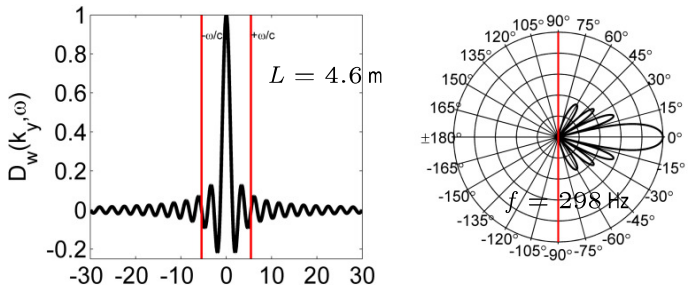
$$D(y_0,\omega) = 1$$

$$D(k_y,\omega) = 2\,\pi\,\delta(k_y)$$



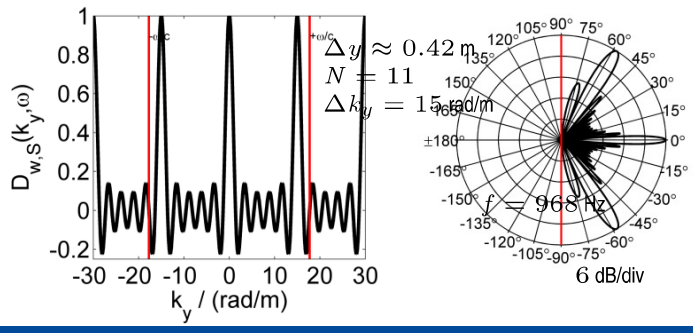
continuous, finite length LSA:

$$D_w(k_y,\omega) = \frac{\sin\left(k_y\,\frac{L}{2}\right)}{k_y\,\frac{L}{2}}$$



discretized, finite length, monopoles LSA:

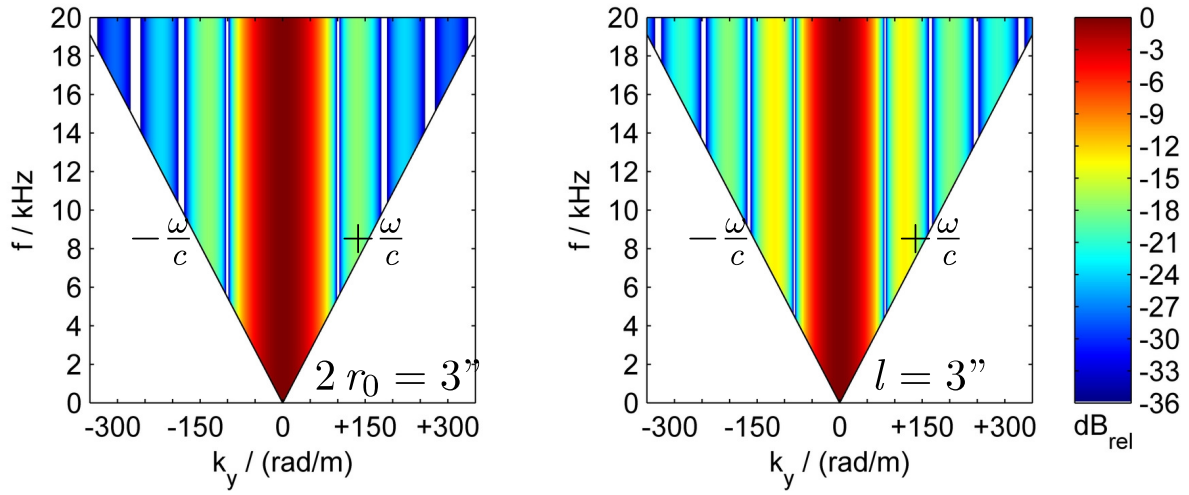
$$D_{w,\text{S}}(k_y,\omega) = \frac{1}{N}\,\frac{\sin(k_y\,\Delta y\,N/2)}{\sin(k_y\,\Delta y/2)}$$



visible region: $|k_y| \leq |\frac{\omega}{c}|$

$2\,\pi\,\delta(\omega - \omega_{\text{CW}})$ omitted in all spectra

Postfilter $H_{\text{Post}}(k_y, \omega)$ →Piston Farfield Directivities



Circular Piston

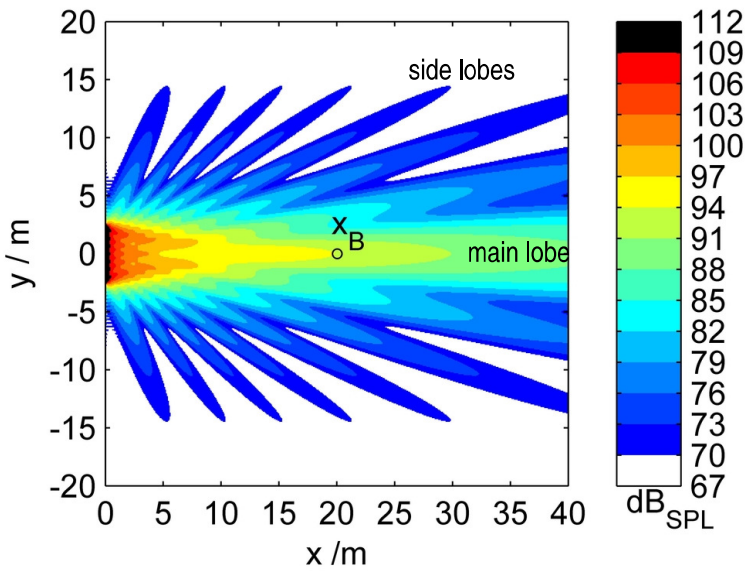
$$H_{\text{Circ}}(k_y, \omega) = \frac{2\,J_1(k_y\,r_0)}{k_y\,r_0} = \frac{2\,J_1(\frac{\omega}{c}\,\sin\varphi\,r_0)}{\frac{\omega}{c}\,\sin\varphi\,r_0}$$

Line Piston

$$H_{\text{Rect}}(k_y, \omega) = \frac{\sin\left(k_y\,\frac{l}{2}\right)}{k_y\,\frac{l}{2}} = \frac{\sin\left(\frac{\omega}{c}\,\sin\varphi\,\frac{l}{2}\right)}{\frac{\omega}{c}\,\sin\varphi\,\frac{l}{2}}$$

angular spectrum vs. polar diagram

Finite Length, Uniformly Driven, Continuous LSA



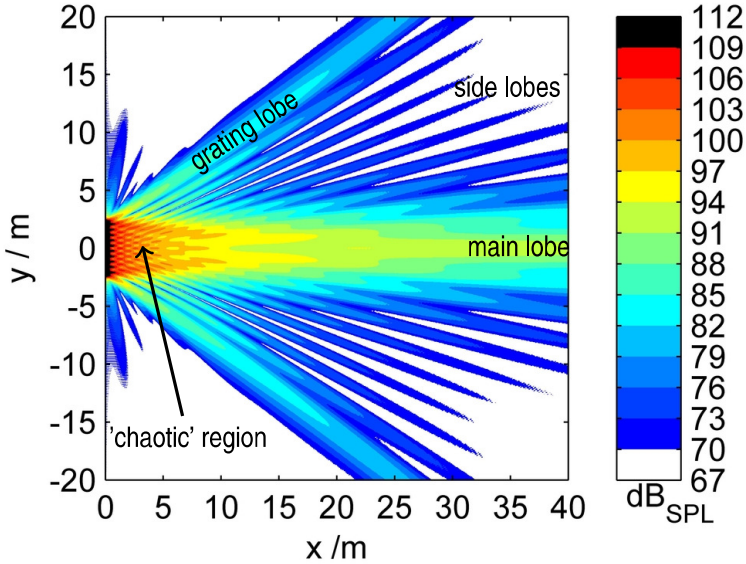
$L = 5.05\,\text{m}$

$f = 8 \cdot c/L = 543.4\,\text{Hz}$

x_B...border distance between Fresnel/Fraunhofer region

$$P(x, y, \omega) = \frac{1}{2\,\pi} \int_{-\infty}^{+\infty} \underbrace{\frac{\sin\left(k_y\,\frac{L}{2}\right)}{k_y\,\frac{L}{2}}}_{D_w(k_y,\omega)} \cdot G_0(x, k_y, \omega) \cdot \mathrm{e}^{-\mathrm{j}\,k_y\,y}\,\mathrm{d}k_y$$

Finite Length, Uniformly Driven, Discretized LSA



$N = 11$ Line Pistons

$L = 4.97\,\text{m}$

$l = 0.381\,\text{m}$

$\Delta y = 0.4591\,\text{m}$

$\text{ARF} = \frac{N \cdot l}{L} = 0.84$

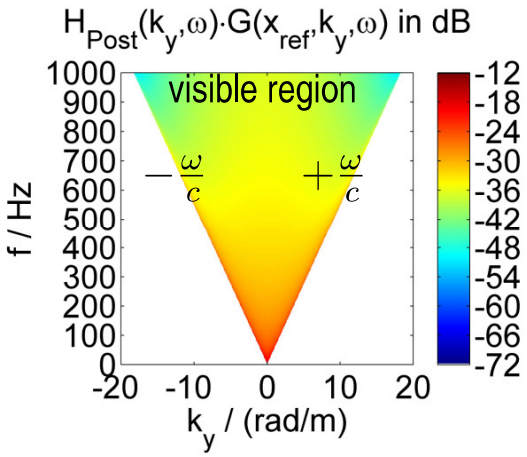
ARF...Active Radiation Factor

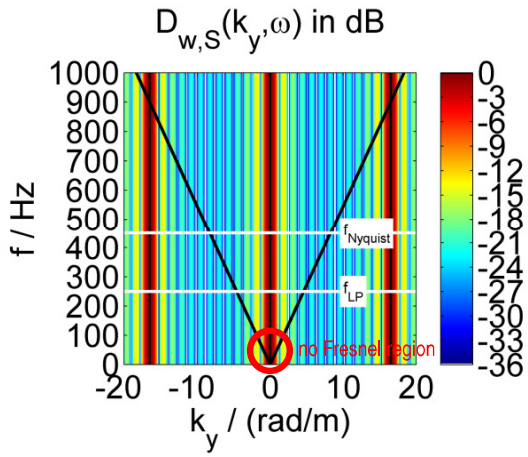
$f = 1.2\,\text{kHz}$

$$P(x,y,\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \overbrace{D_{w,\text{S}}(k_y,\omega)\, H_{\text{post}}(k_y,\omega)}^{D_{w,\text{S,H}}(k_y,\omega)}\, G_0(x,k_y,\omega)\, \mathrm{e}^{-\mathrm{j}\, k_y\, y}\, \mathrm{d}k_y$$

$$D_{w,\text{S}}(k_y,\omega) = \frac{1}{N} \frac{\sin(k_y\, \Delta y\, N/2)}{\sin(k_y\, \Delta y/2)} \qquad H_{\text{Rect}}(k_y,\omega) = \frac{\sin\left(k_y\, \frac{l}{2}\right)}{k_y\, \frac{l}{2}}$$

Line Source Array with 15" Circular Pistons (WST #2)





$L = 4.95\,\text{m}$

$\text{ARF} = \frac{\pi\, r_0^2}{(2\, r_0)^2} = \pi/4$

$N = 13$ circular pistons

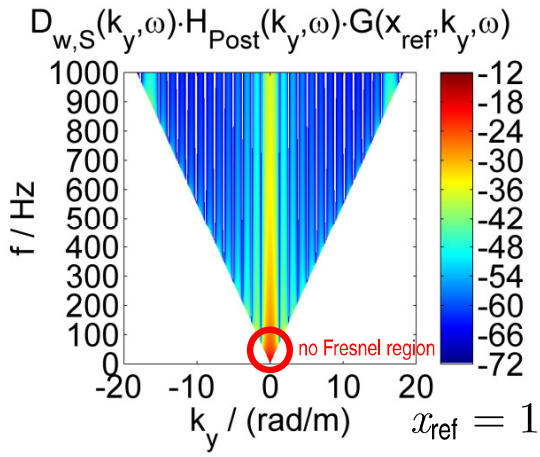
$\Delta y = 2\, r_0 = 15" = 0.381\,\text{m}$

$\Delta k_y = \frac{2\,\pi}{\Delta y} = 16.49\,\text{rad/m}$

typical temporal lowpass @ $\approx 250\,\text{Hz}$

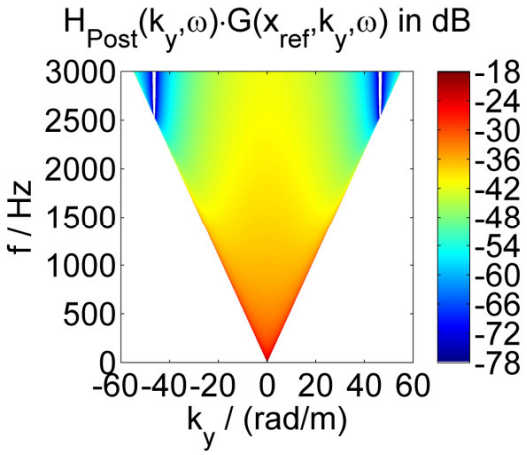
sampling theorem $\hat{=}$ WST #2:

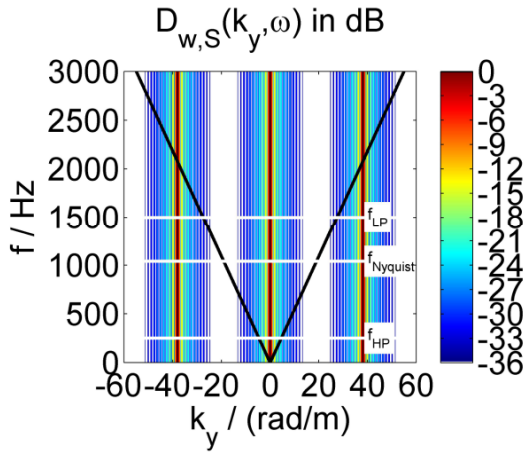
$k_{y,\text{Nyquist}} = \frac{\pi}{\Delta y} \leftrightarrow f_{\text{Nyquist}}$



$x_{\text{ref}} = 10\,\text{m}$

Line Source Array with 6.5" Circular Pistons (WST #2)





$L = 4.95$ m

ARF $= \pi/4$

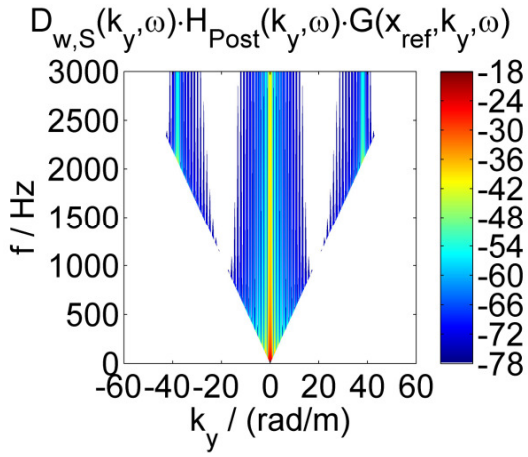
$N = 30$ circular pistons

$\Delta y = 2\, r_0 = 6.5" = 0.1651$ m

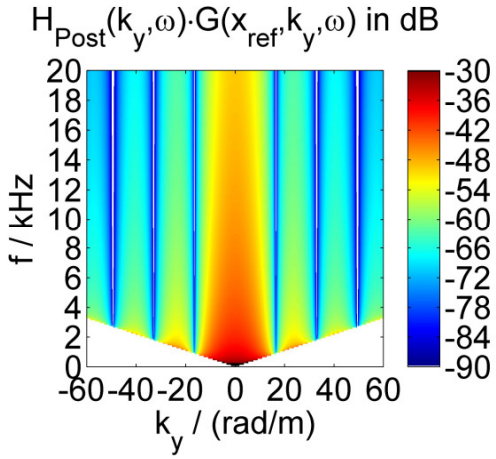
$\Delta k_y = \frac{2\,\pi}{\Delta y} = 38.06$ rad/m

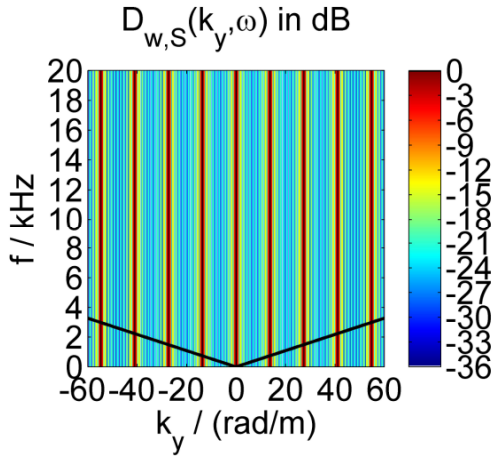
typical temporal bandpass @

≈ 250 Hz & 1.5 kHz



Line Source Array with Line Pistons, ARF=0.84 (WST #1)





$L = 4.97$ m

ARF $= 0.84$

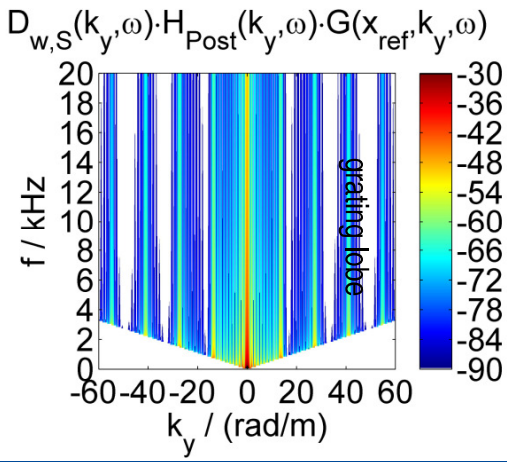
$N = 11$ line pistons

$l = 0.381$ m

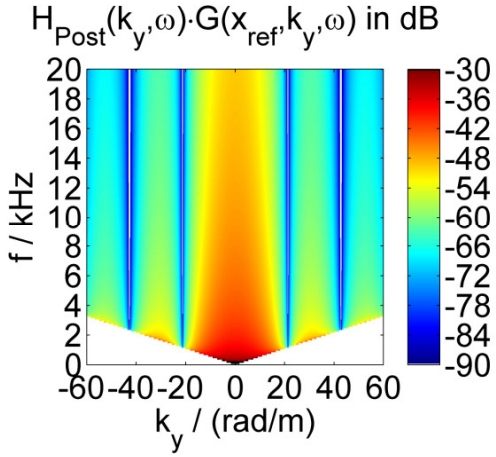
$\Delta y = 0.4591$ m

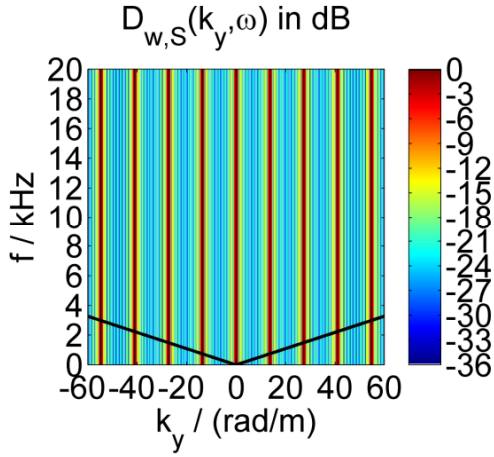
$\Delta k_y = \frac{2\,\pi}{\Delta y} = 13.69$ rad/m

WST #1: grating lobes are tolerated, use spatial lowpass characteristics of pistons



Line Source Array with Line Pistons, ARF=0.66 ~~(WST #1)~~





$L = 4.89$ m

ARF $= 0.66$

11 Pistons

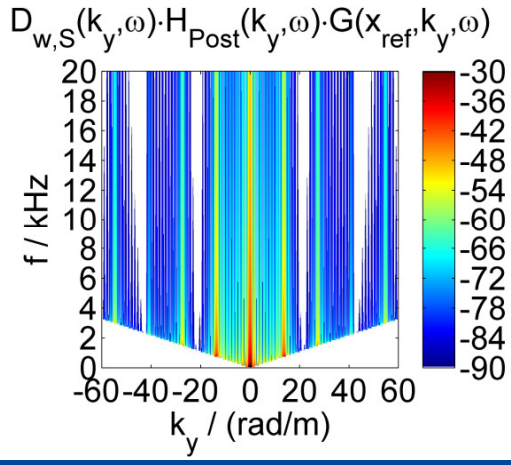
$l = 0.294$ m

$\Delta y = 0.4591$ m

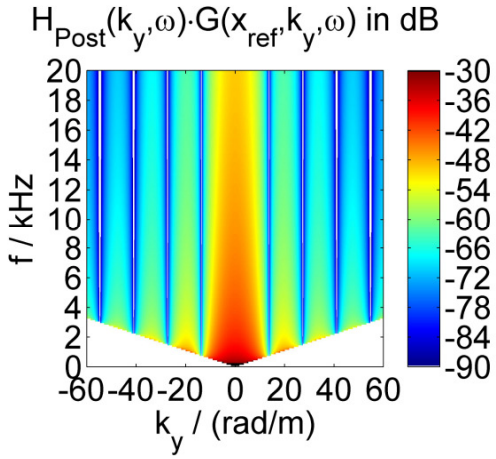
$\Delta k_y = \frac{2\pi}{\Delta y} = 13.67$ rad/m

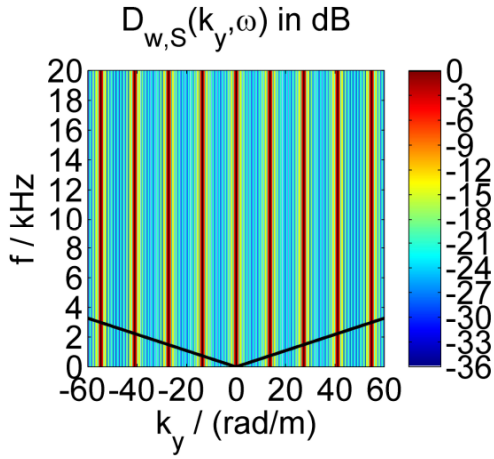
typical temporal highpass @ ≈ 1.5 kHz

grating lobe amplitudes > -13.5 dB$_{\text{rel}}$ occur



Line Source Array with Line Pistons, ARF=1 (WST #1)





$L = 5.05$ m

ARF $= 1$

$N = 11$ line pistons

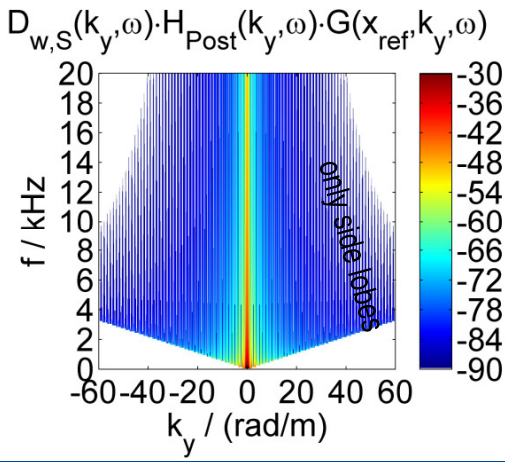
$\Delta y = l = 0.4591$ m

$\Delta k_y = \frac{2\pi}{\Delta y} = 13.69$ rad/m

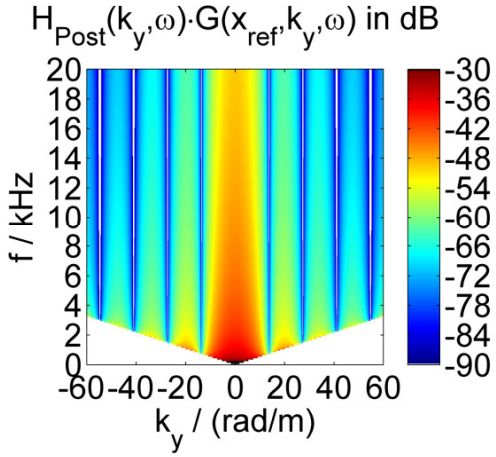
typical temporal highpass @ ≈ 1.5 kHz

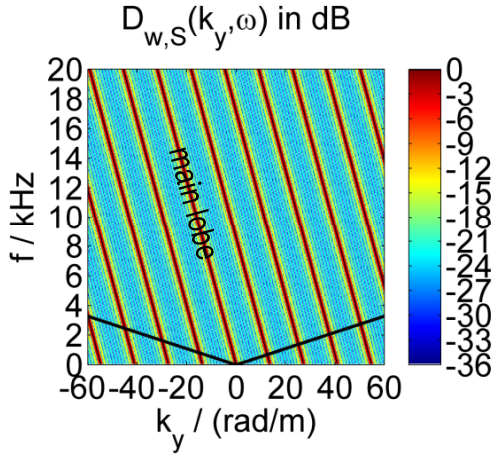
special case here: perfect aliasing suppression

$D_w(k_y, \omega) = D_{w,\text{S}}(k_y, \omega) \cdot H_{\text{Rect}}(k_y, \omega)$



Electronic Beamsteering $\phi = -5°$, ARF=1





$L = 5.05$ m

ARF $= 1$

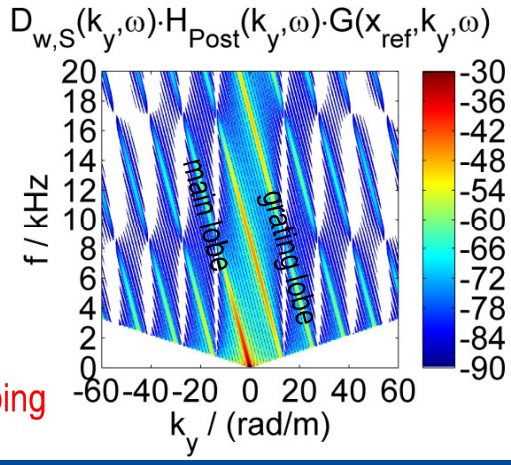
$N = 11$ line pistons

$\Delta y = l = 0.4591$ m

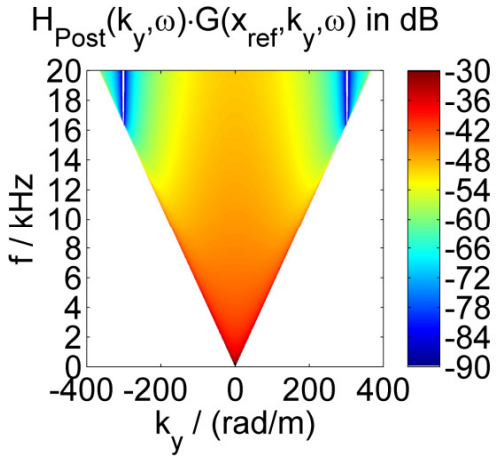
$\Delta k_y = \frac{2\pi}{\Delta y} = 13.69$ rad/m

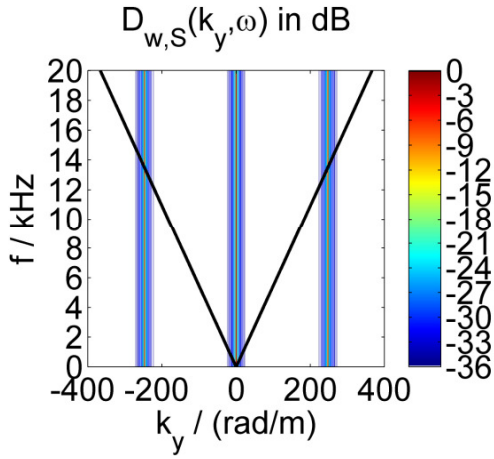
cf. right figure on slide 2

electronic beamsteering with large line pistons not useful→ only geometric and amplitude beamshaping



Line Source Array with 1" Circular Pistons





$L = 4.95$ m

ARF $= \pi/4$

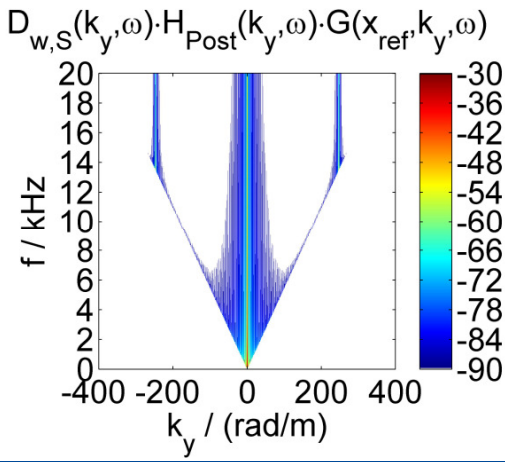
$N = 195$ circular pistons

$\Delta y = 1" = 0.0254$ m

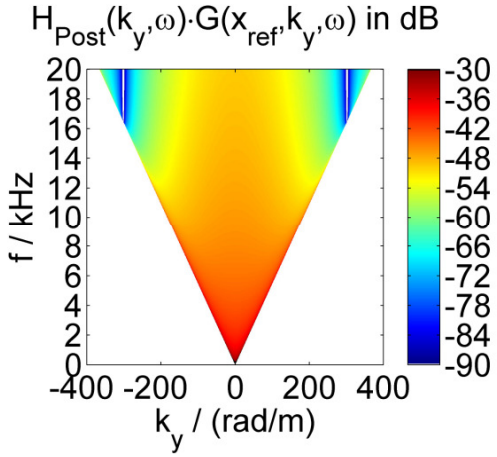
$\Delta k_y = \frac{2\pi}{\Delta y} = 247.37$ rad/m

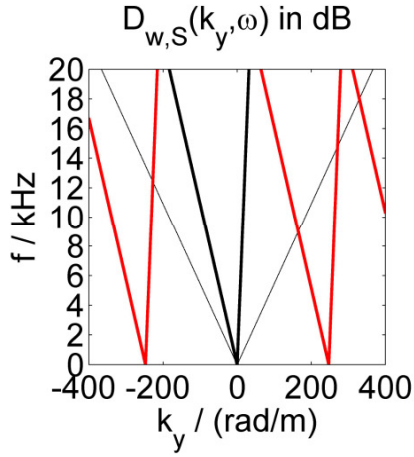
typical temporal highpass @ ≈ 1.5 kHz

very fine granularity→spatial aliasing problem only in very high temporal frequencies



Electronic Beamforming And -Steering, Circular Pistons





$L = 4.95$ m

ARF $= \pi/4$

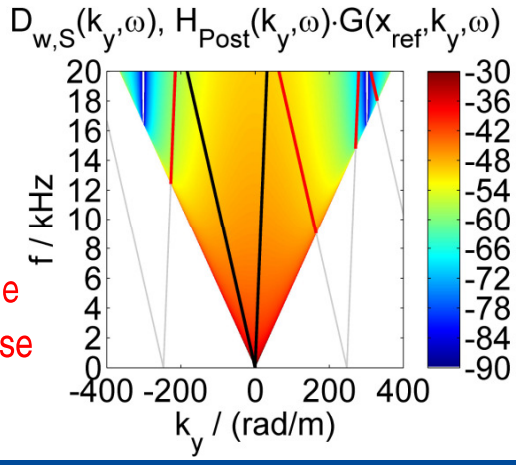
$N = 195$ circular pistons

$\Delta y = 2\,r_0 = 1" = 0.0254$ m

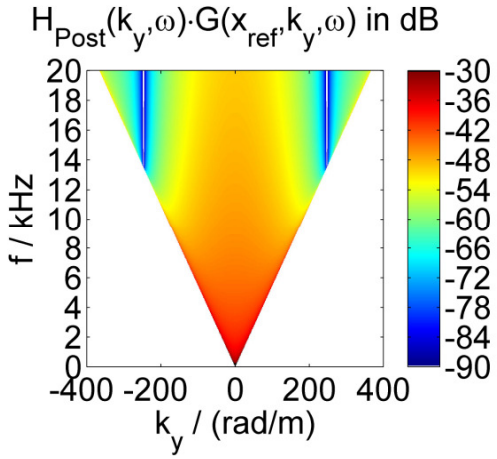
$\Delta k_y = \frac{2\,\pi}{\Delta y} = 247.37$ rad/m

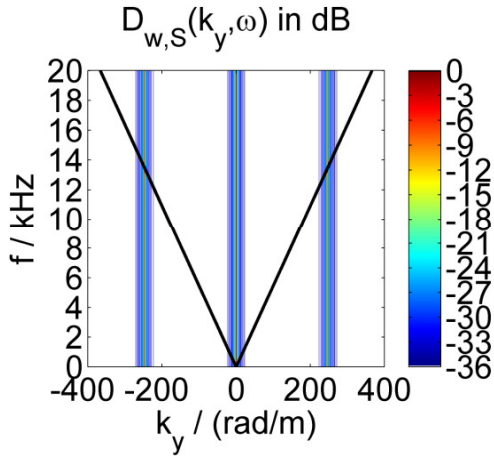
constant farfield directivity of $35°$ $\rightarrow$ black triangle

trade-off between max SPL and frequency response variation



Line Source Array with 1" Line Pistons, ARF=1





$L = 4.95$ m

ARF $= 1$

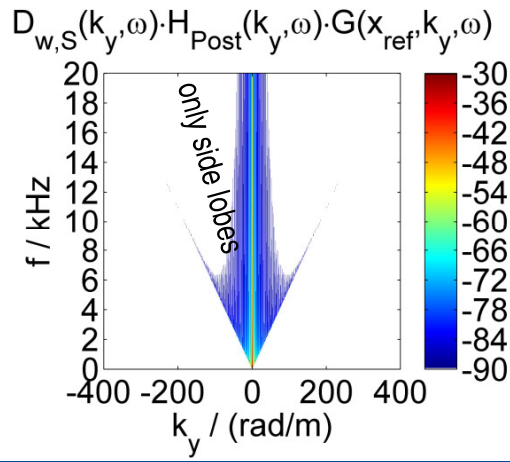
195 Pistons

$\Delta y = l = 1" = 0.0254$ m

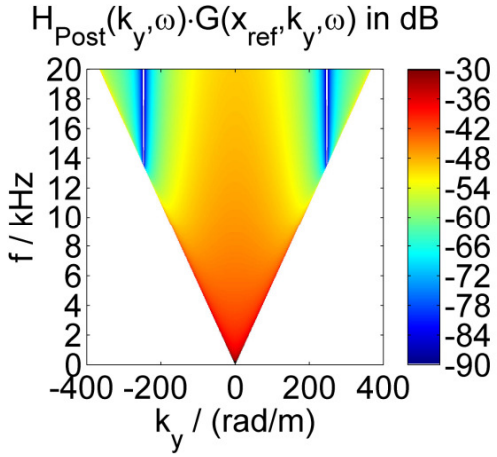
$\Delta k_y = \frac{2\,\pi}{\Delta y} = 247.37$ rad/m

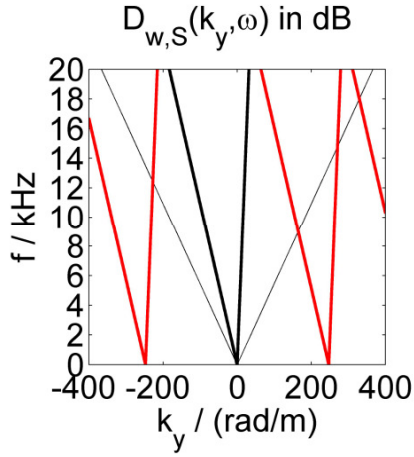
typical temporal highpass @ ≈ 1.5 kHz

very fine granularity$\rightarrow$special case here: perfect aliasing suppression



Electronic Beamforming And -Steering, Line Pistons





$L = 4.95$ m

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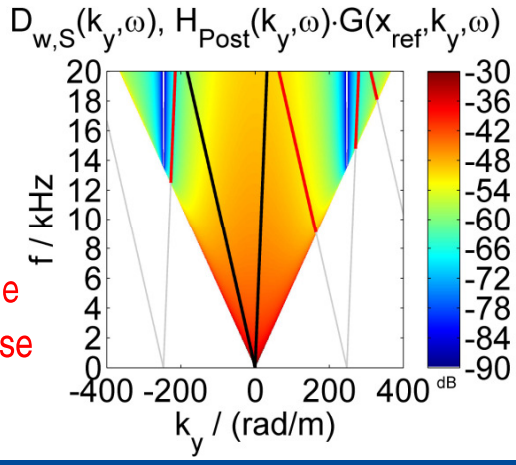
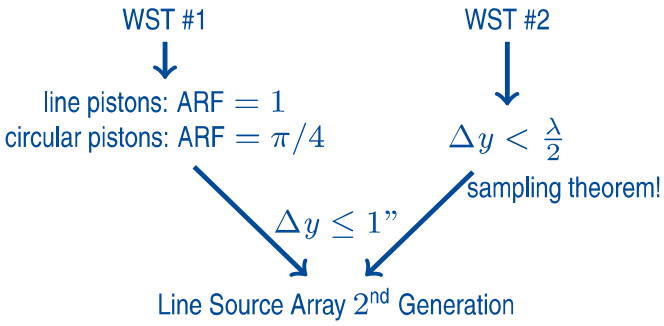


Conclusion

- WST criteria #1 & #2 explained in the spatio-temporal Fourier spectrum domain
- grating lobes (spatial aliasing) must be suppressed by the spatial lowpass reconstruction filter (WST #1) *or* must not enter the visible region of the LSA (WST #2)
- loudspeakers act as non-ideal spatial lowpass filters



- small discretization step $\Delta y \leq 1$cm $\rightarrow$ small drivers$\rightarrow$ non-directed (ideally monopole-like)$\rightarrow$ full and aliasing-free control of the visible region for audio frequencies (≤ 16 kHz) $\rightarrow$ electronic and/or geometric beamforming and -steering$\rightarrow$numerical optimization schemes

slides of this talk available @ `http://spatialaudio.net/`

Schultz, F.; Rettberg, T.; Spors, S. (2014): "On Spatial-Aliasing-Free Sound Field Reproduction using Finite Length Line Source Arrays". In *Proc. of 137th Audio Eng. Soc. Conv., Los Angeles*, #9098.