Universität Rostock — Traditio et Innovatio

Institute of Communications Engineering

Discussion of the Wavefront Sculpture Technology Criteria for Straight Line Arrays

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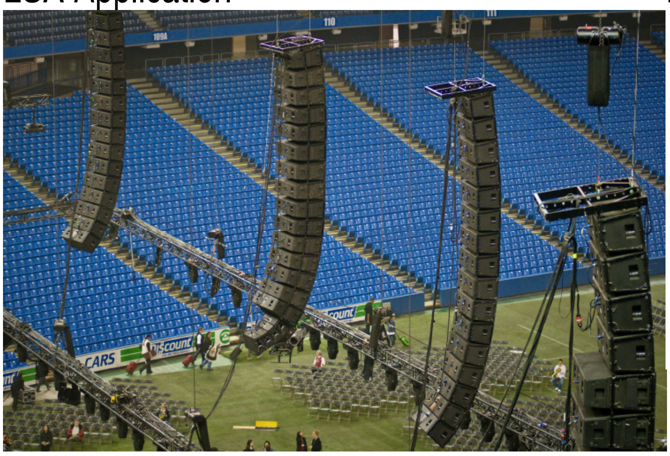
2015-05-10 10:30 a.m., *Spatial Audio Part 2*, P15-4, #9323

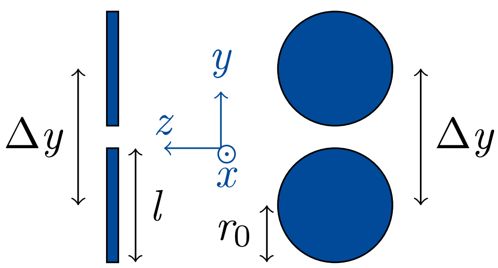
Introduction Line Source Arrays (LSA)

LSA Application



LSA Cabinet

line pistons   circular pistons



$q = \frac{l}{\Delta y}$ $q = \frac{2\,r_0}{\Delta y}$

Wavefront Sculpture Technology® (WST)
[Heil et al., 1992, 92nd AES Conv., Vienna]
[Urban et al., 2003, JAES **51**(10):912-932]

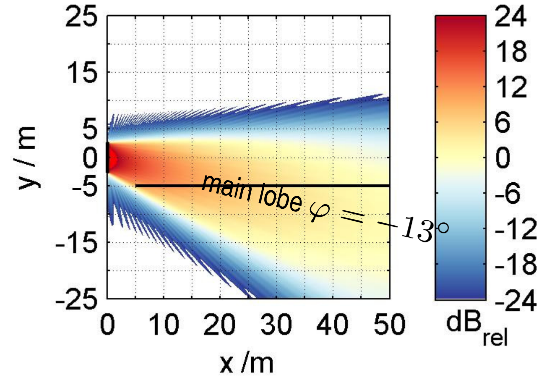
http://www.jblpro.com/press/jan09/JBL_IntlVerTec.jpg
http://www.turbosound.com/public/images/product_zoom/TFS-900H_front.jpg

Version: 0.00 2015-04-29

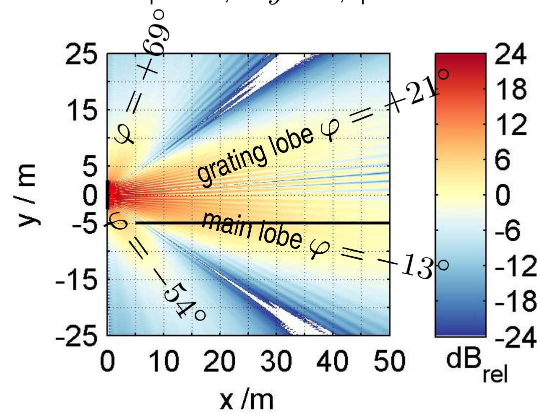
Spatial Aliasing f=3kHz, array length $L \approx 5$m

sound field within xy-plane

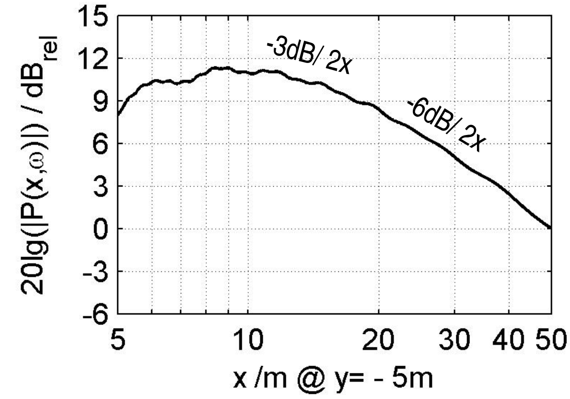
195 × 1" line pistons, Δy=1", q=1

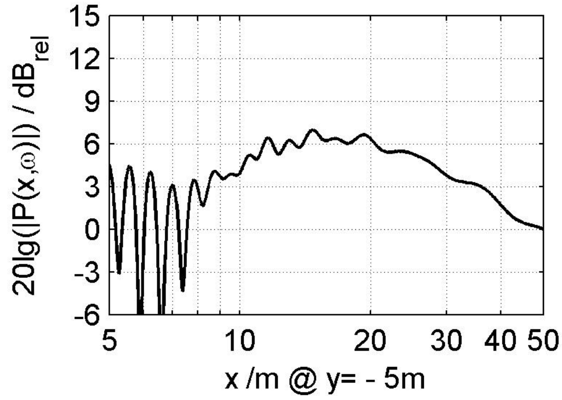


25 × 7.8" line pistons, Δy=7.8", q=1



pressure along x for $y = -5$ m





Angular Spectrum Synthesis

based on the inverse spatial Fourier transform using propagating waves only

$k_y = \frac{\omega}{c} \sin\varphi \;\circ\!\!-\!\!\bullet\; y$

$$P(x, y, \omega) = \frac{1}{2\pi} \int_{-\pi/2}^{+\pi/2} \left[D_{w,\mathrm{S}}(\varphi, \omega) \cdot H_{\mathrm{Post}}(\varphi, \omega)\right] \times \left[\frac{-\mathrm{j}}{4} H_0^{(2)}\left(\frac{\omega}{c}\cos\varphi\, x\right) \times \mathrm{e}^{-\mathrm{j}(\frac{\omega}{c}\sin\varphi\, y)}\right] \frac{\omega}{c}\cos\varphi\, \mathrm{d}\varphi$$

$D_{w,\mathrm{S}}(\varphi, \omega) \propto$
angular spectrum of source's velocity along the y-axis, i.e. **array factor**

$H_{\mathrm{Post}}(\varphi, \omega)$
angular spectrum of acoustic postfilter, i.e. loudspeaker **farfield radiation pattern**

$D_{w,\mathrm{S}}(\varphi, \omega) \cdot H_{\mathrm{Post}}(\varphi, \omega)$
final array factor, i.e the farfield radiation pattern of the line array

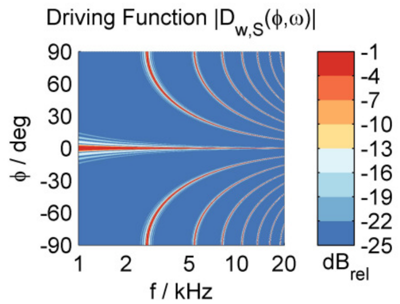
Angular Spectrum Synthesis

based on the inverse spatial Fourier transform using propagating waves only

$k_y = \frac{\omega}{c}\sin\varphi$ ○—● y

$$P(x, y, \omega) = \frac{1}{2\pi} \int\limits_{-\pi/2}^{+\pi/2} \left[D_{w,\mathrm{S}}(\varphi,\omega) \cdot H_{\mathrm{Post}}(\varphi,\omega)\right] \times \left[\frac{-\mathrm{j}}{4} H_0^{(2)}\left(\frac{\omega}{c}\cos\varphi\, x\right) \times \mathrm{e}^{-\mathrm{j}(\frac{\omega}{c}\sin\varphi\, y)}\right] \frac{\omega}{c}\cos\varphi\,\mathrm{d}\varphi$$

for each evaluation point $\vec{x} = (x, y)^T$:



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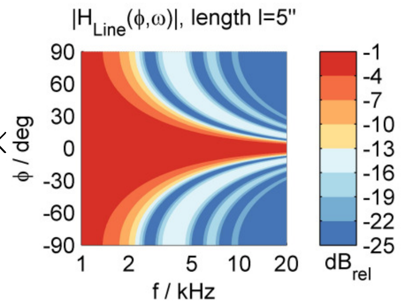

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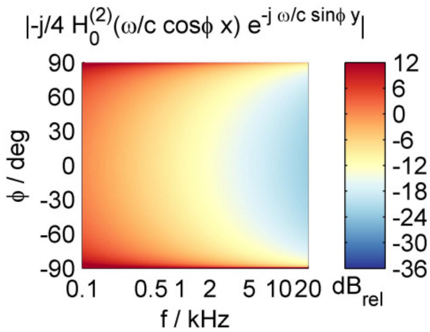
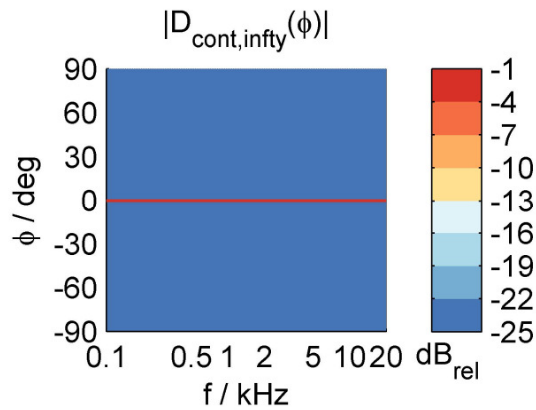


and then $\int \ldots \mathrm{d}\varphi$

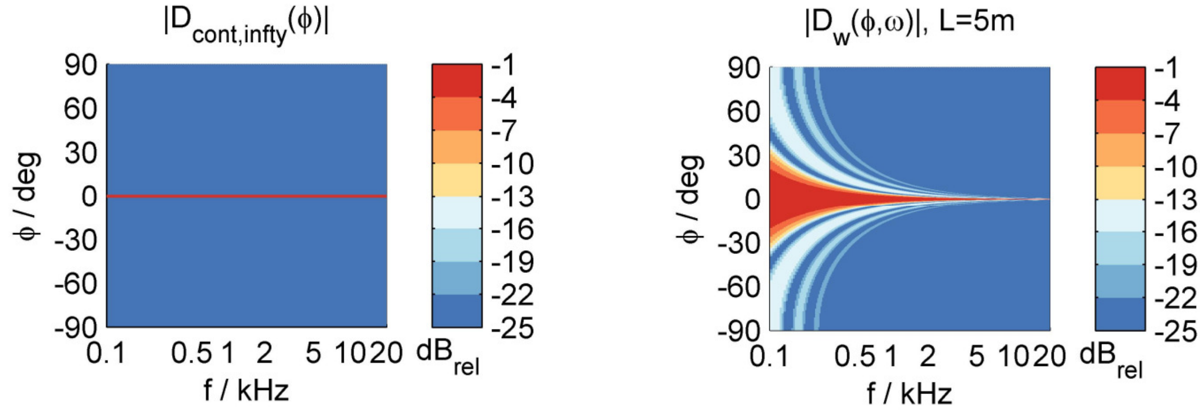
Angular Spectrum of Source Velocity, WST Reference

infinite, continuous LSA



Angular Spectrum of Source Velocity, WST Reference

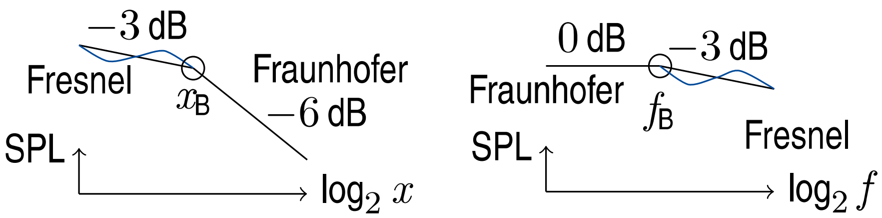
infinite, continuous LSA vs. finite length, continuous, rect. win LSA with length L



-Fresnel / Fraunhofer region $x_\mathrm{B}(l, f)$

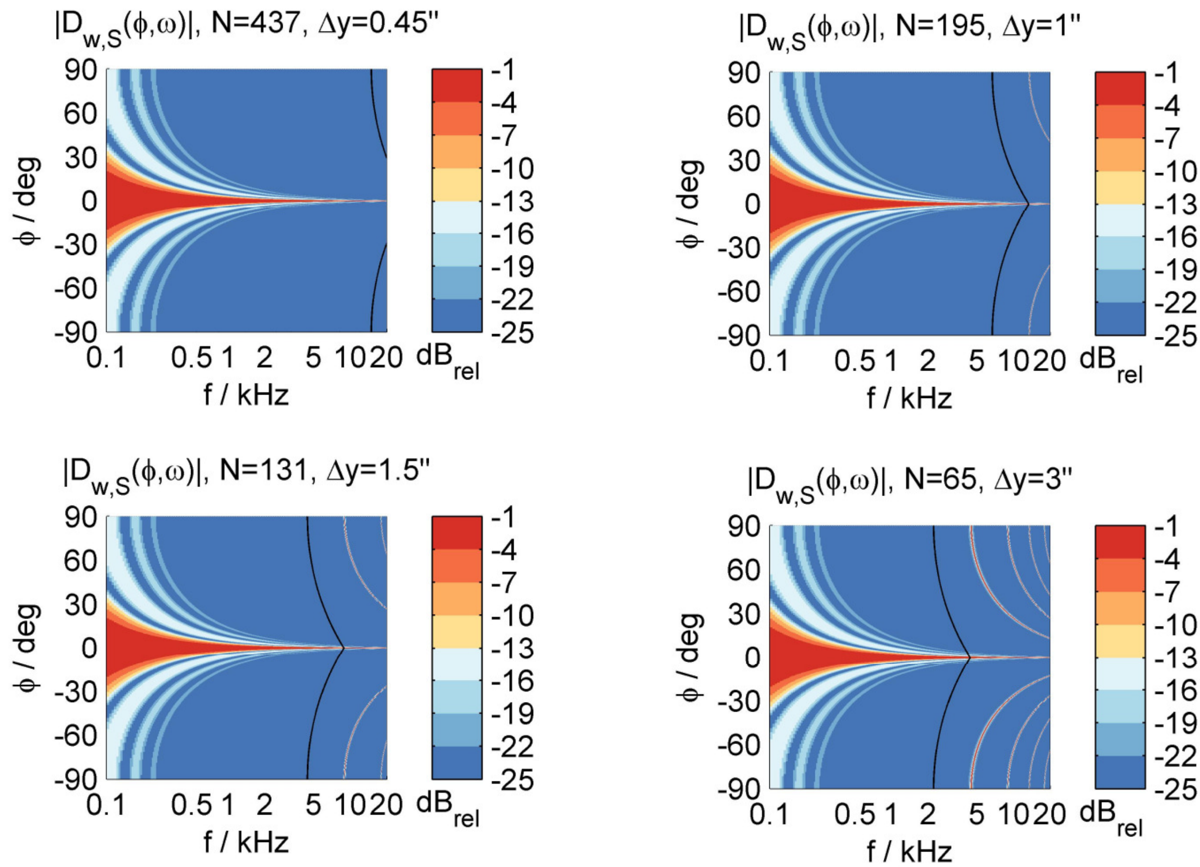
-farfield radiation pattern highly dependent from L and f

-ripples in Fresnel region (window, spatial aliasing)



Angular Spectrum of Source Velocity, Discretization

$L = \Delta y\, N \approx 5$ m, finite length, discretized LSA with ideal point sources



Angular Spectrum of Source Velocity, Discretization

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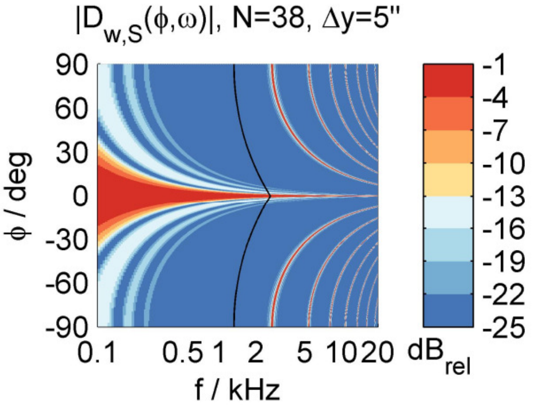
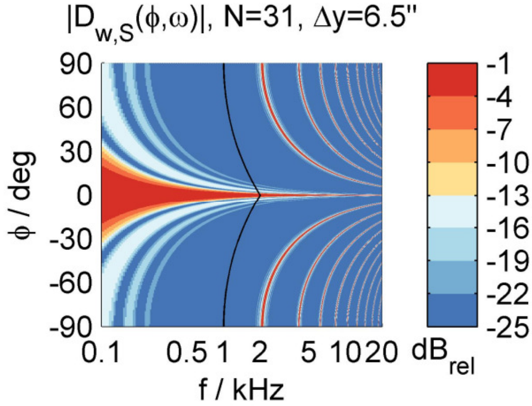
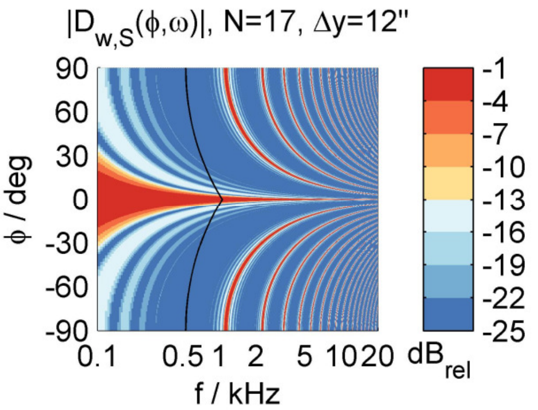
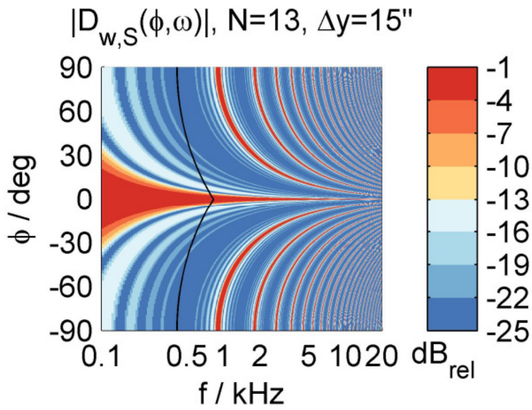


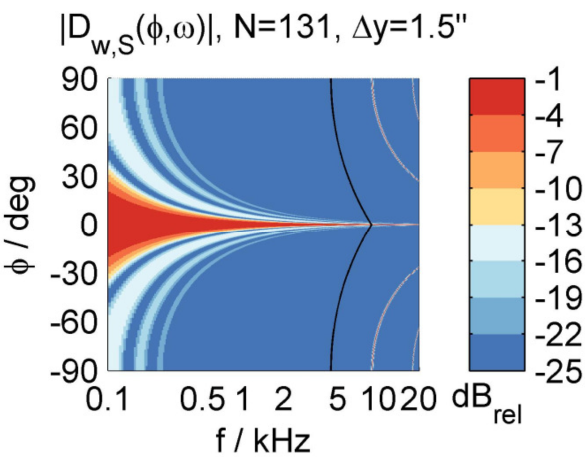


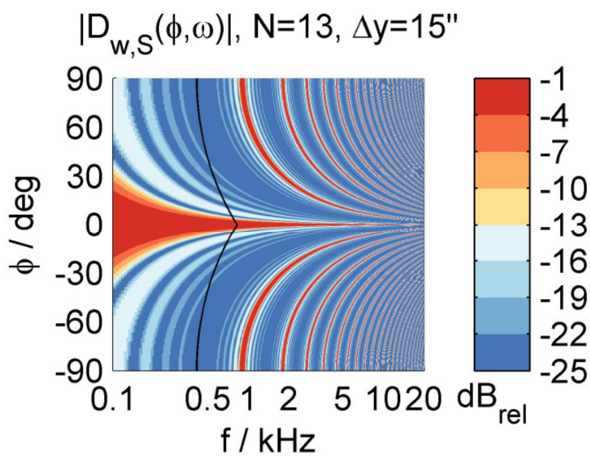




Discretization, HF-Band Problem

required sampling at high frequencies $\rightarrow$ sampling condition $\Delta y < \lambda_{\text{min}}/2$, i.e. WST#2

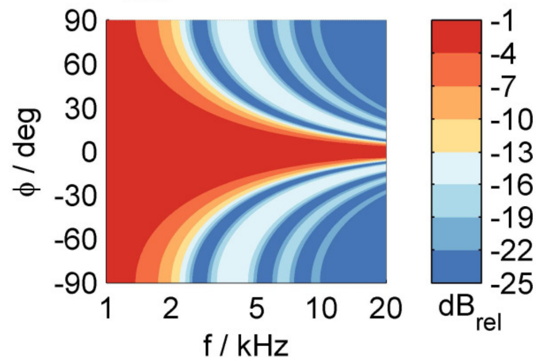




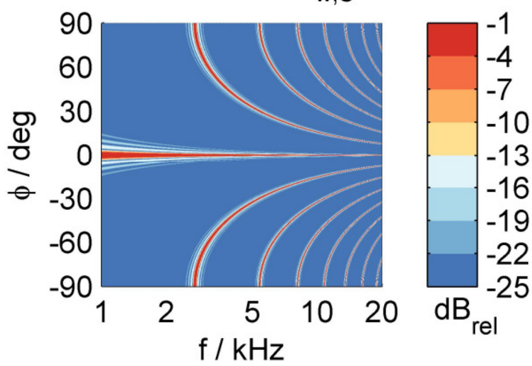
LSA with Pistons, No Gaps between Pistons

loudspeaker's farfield radiation pattern acts as spatial lowpass, here $H_{\text{Post}} = H_{\text{Line}}$

$|H_{Line}(\phi,\omega)|$, length l=5"



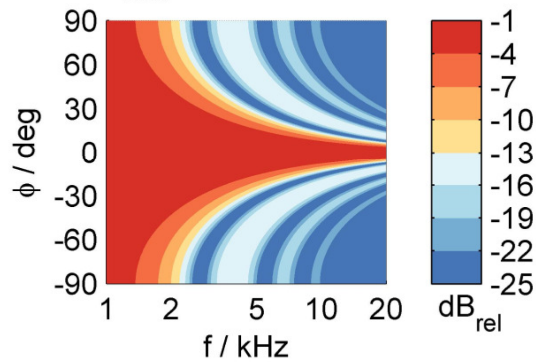
Driving Function $|D_{w,S}(\phi,\omega)|$



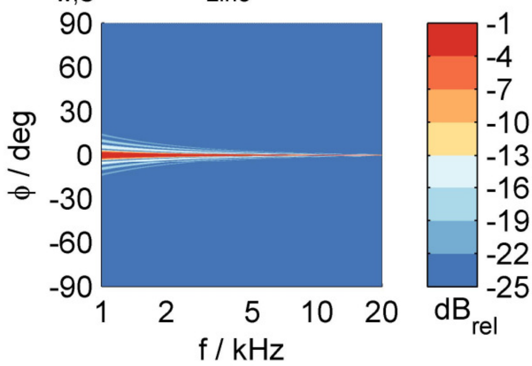
LSA with Pistons, No Gaps between Pistons

perfect reconstruction towards $D_w(\phi, \omega)$ with line piston of length l if $\Delta y = l$

$|H_{Line}(\phi,\omega)|$, length l=5"

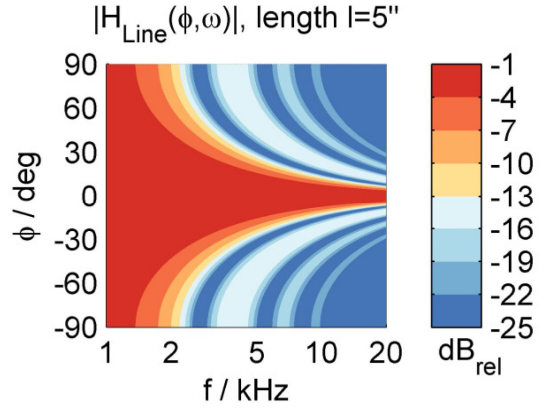


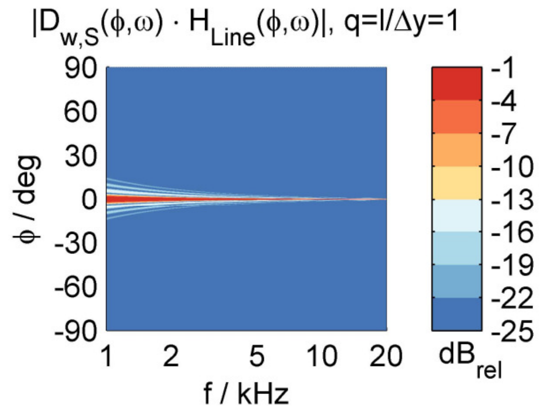
$|D_{w,S}(\phi,\omega) \cdot H_{Line}(\phi,\omega)|$, q=l/Δy=1

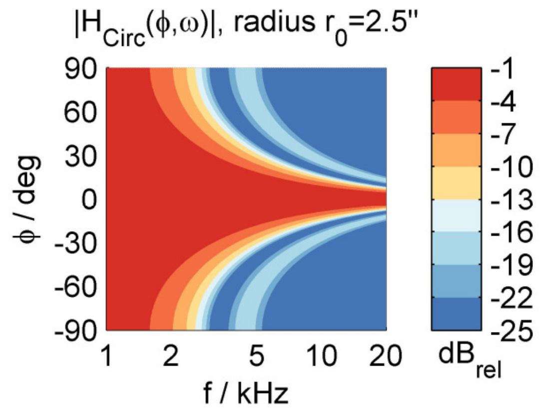


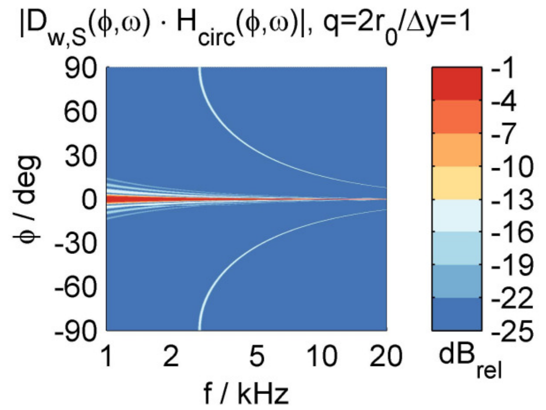
LSA with Pistons, No Gaps between Pistons

line pistons (length l) vs. circular pistons (radius r_0), $\Delta y = l = 2\,r_0$



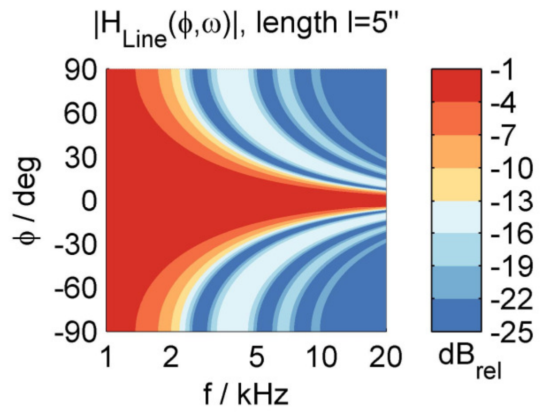


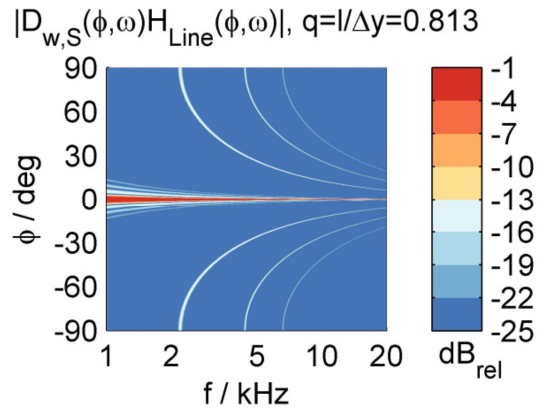




WST#1 Criterion, >13.26 dB Grating Lobe Attenuation

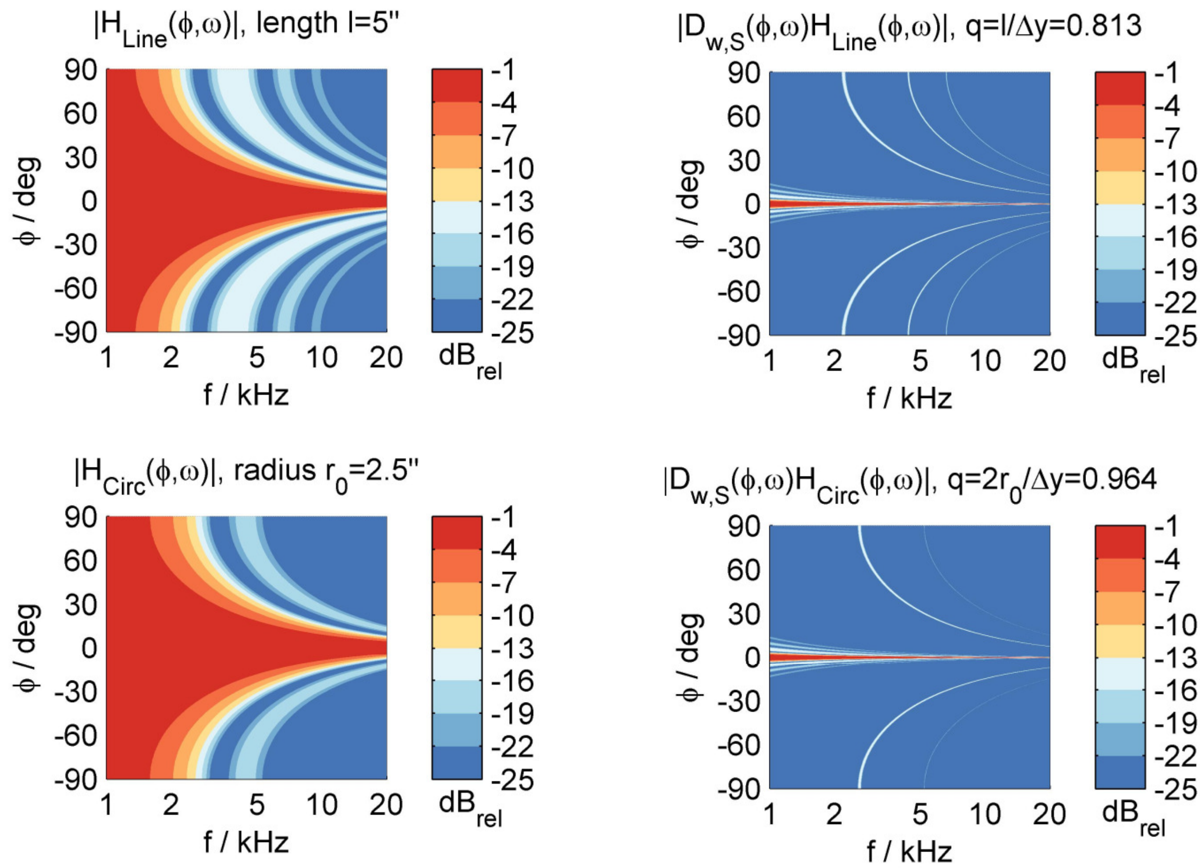
aka the Active Radiating Factor (ARF) criterion





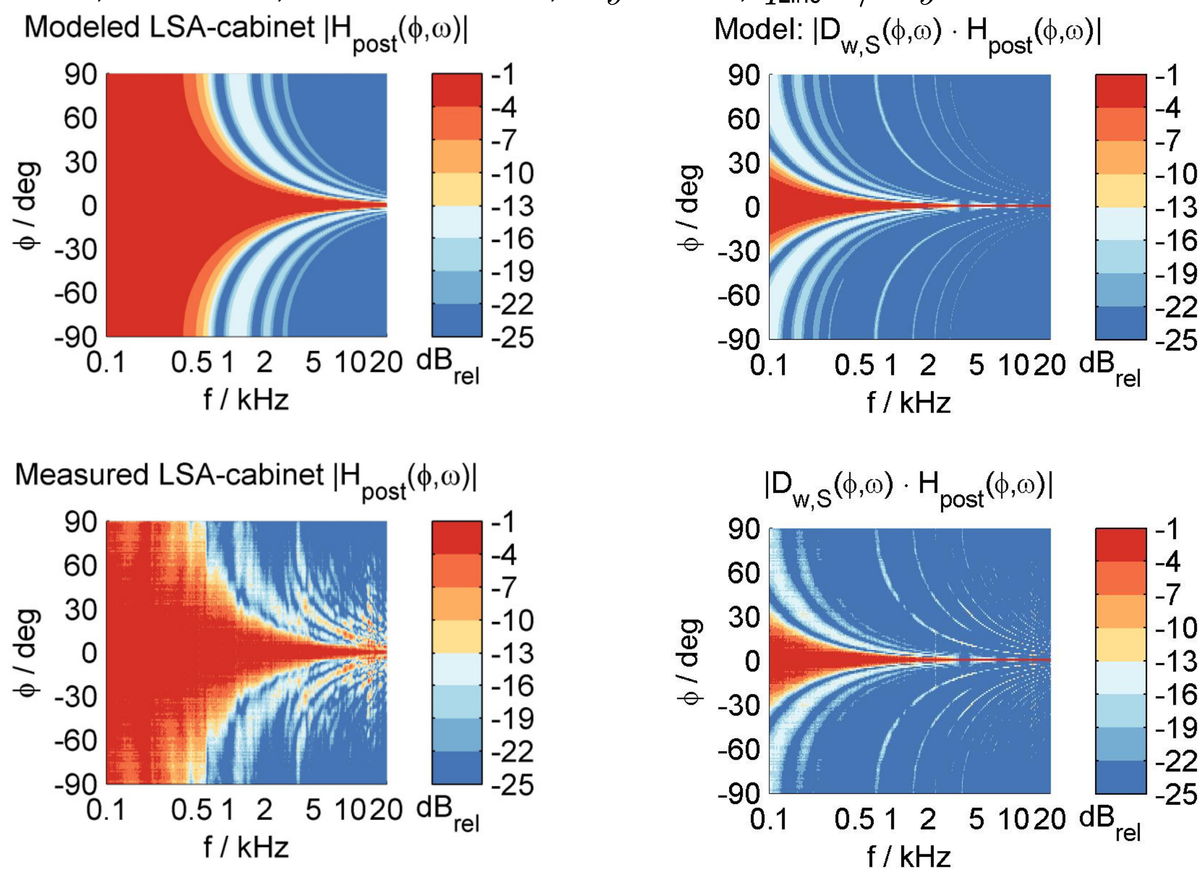
WST#1 Criterion, >13.26 dB Grating Lobe Attenuation

WST#1 criterion can be derived for both, the line and the circular piston



Model vs. Practical Example

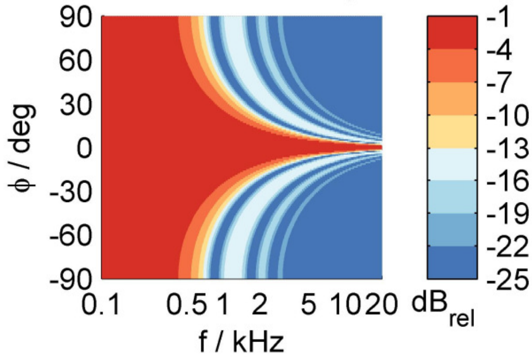
N=11, LF: 15" circ, 16.05" line MF/HF, Δy=17.83", $q_{Line}=l/\Delta y$=0.9



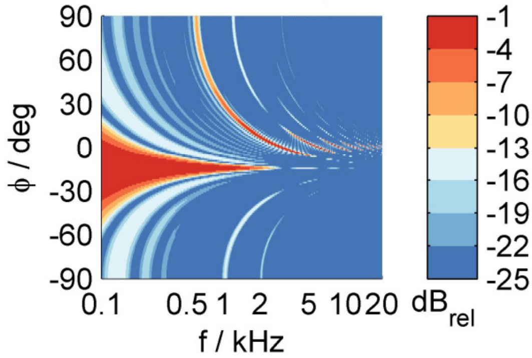
Model vs. Practical Example, $\varphi_{\text{Steer}} = -15°$

beamsteering not meaningful due to too large Δy=17.83"=0.453 m$\rightarrow$ sampling theorem :-(

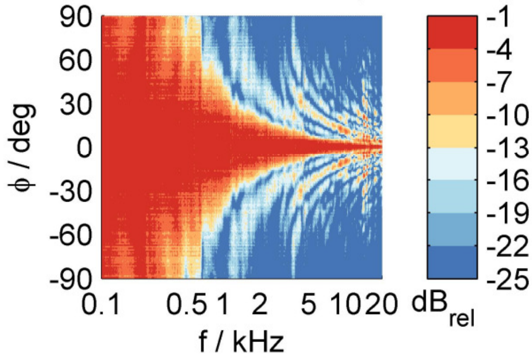
Modeled LSA-cabinet $|H_{post}(\phi,\omega)|$



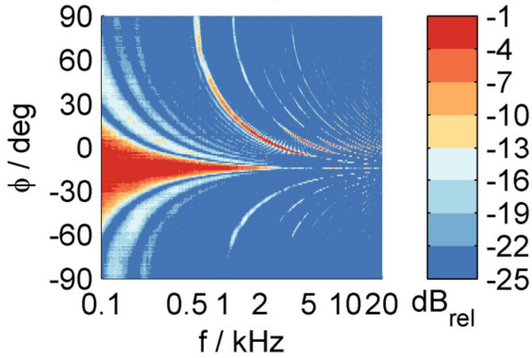
Model: $|D_{w,S}(\phi,\omega) \cdot H_{post}(\phi,\omega)|$



Measured LSA-cabinet $|H_{post}(\phi,\omega)|$



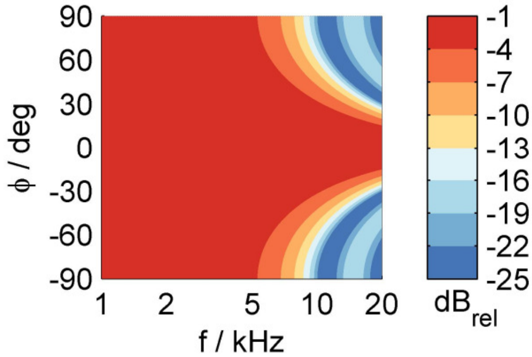
$|D_{w,S}(\phi,\omega) \cdot H_{post}(\phi,\omega)|$



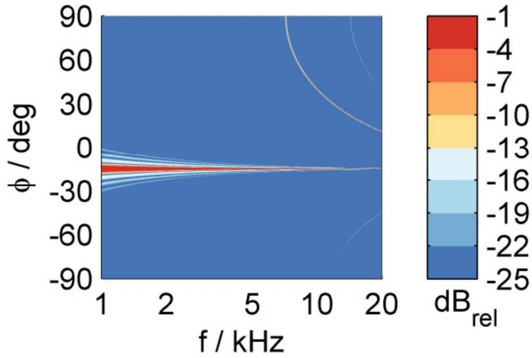
2nd Generation LSAs $\rightarrow$ (Almost) Full WST#2 Compliant

very small discretization step, very small pistons$\rightarrow$ WST#1 less important

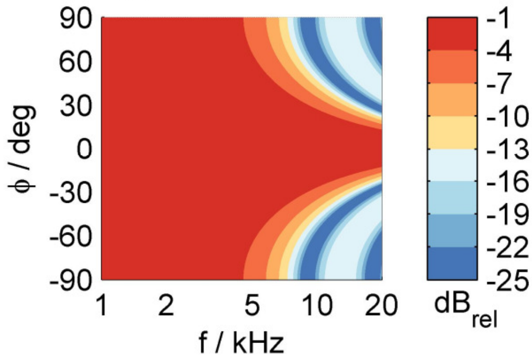
$|H_{Circ}(\phi,\omega)|$, radius r_0=0.75"



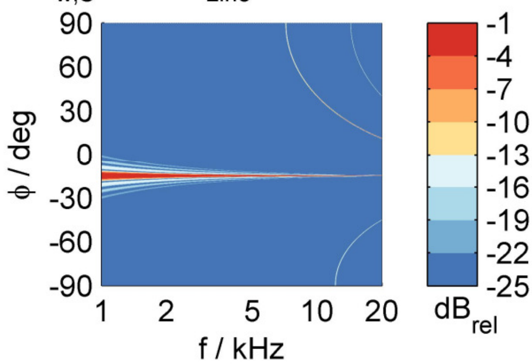
$|D_{w,S}(\phi,\omega) \cdot H_{Circ}(\phi,\omega)|$, $q=2r_0/\Delta y=1$



$|H_{Line}(\phi,\omega)|$, length l=1.5"

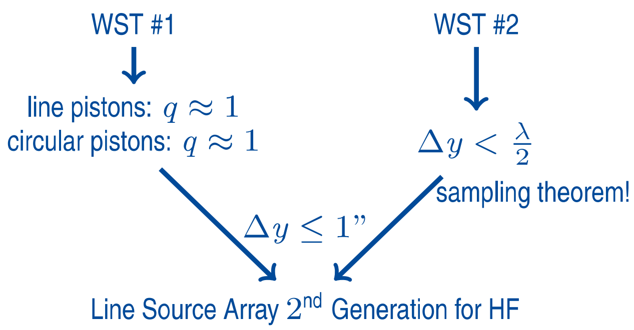


$|D_{w,S}(\phi,\omega) \cdot H_{Line}(\phi,\omega)|$, $q=l/\Delta y=1$



Conclusion

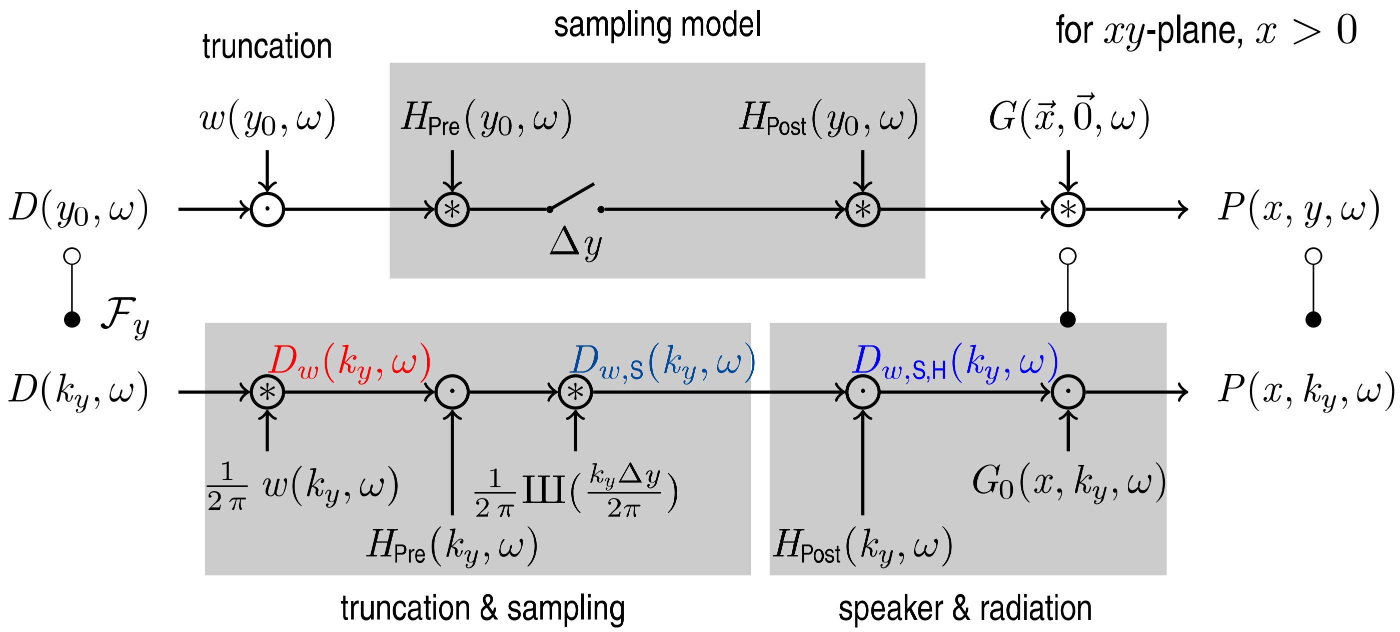
- WST criteria #1, #2, #3 discussed in the spatio-temporal Fourier spectrum domain
- grating lobes (spatial aliasing) must be suppressed by the spatial lowpass reconstruction filter (WST #1) *or* must not enter the visible region of the LSA (WST #2)
- loudspeakers act as non-ideal spatial lowpass filters (WST #3)
- WST#1, #3 and #3 interact



slides & PhD-chapter draft available @ `http://spatialaudio.net/`

Schultz, F.; Straube, F.; Spors, S. (2015): "Discussion of the Wavefront Sculpture Technology Criteria for Straight Line Arrays". In *Proc. of 138th Audio Eng. Soc. Conv., Warsaw*, #9323.

Signal Processing Model for Sound Field Synthesis



cf. [Start, 1997, PhD]

farfield radiation pattern in **visible** region $|k_y| \leq |\frac{\omega}{c}|, -90° \leq \phi \leq +90°$

$D_w(k_y, \omega)$ ...continuous, finite length LSA

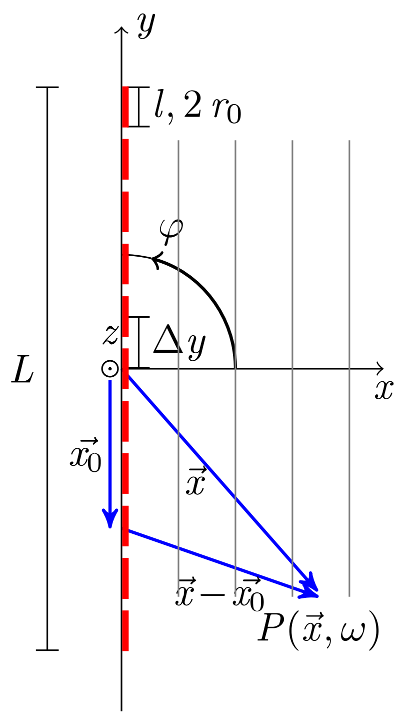
$D_{w,\text{S}}(k_y, \omega)$ ...discretized, finite length, point source LSA (array factor)

$D_{w,\text{S,H}}(k_y, \omega)$...discretized, finite length, loudspeaker LSA (1st product theorem, final array factor)

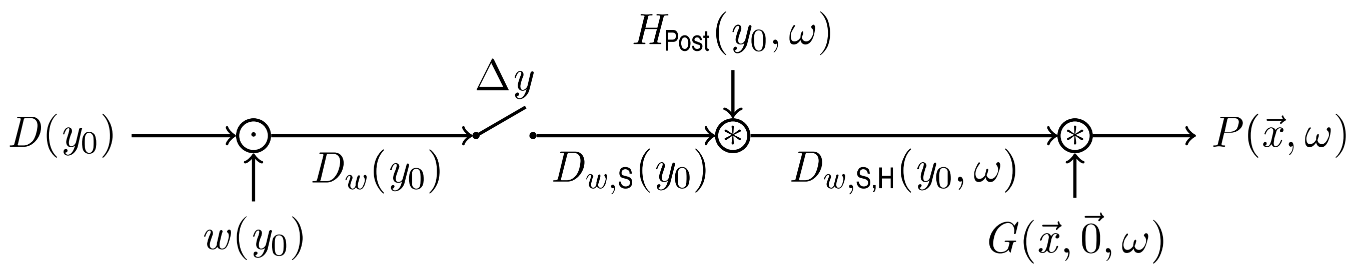
Wavefront Sculpture Technology (WST)

WST considers a uniform linear array (ULA) without beam steering, either with ideal point sources or with identical circular / line pistons aiming at:

1. reducing or avoiding spatial aliasing
   - with **WST #2**, i.e. by fulfilling the spatial sampling theorem, $\Delta y < \frac{\lambda_{\text{min}}}{2}$
   - with **WST #1**, i.e. with loudspeaker's spatial lowpass by controlling $q = \frac{l}{\Delta y}$, $q = \frac{2\,r_0}{\Delta y}$
   - with **WST #3**, i.e. with loudspeaker's spatial lowpass by controlling the line piston's velocity
2. homogeneous coverage over the audience with additional array curving
   - **WST #4**: frequency independent near/farfield transition
   - **WST #5**: maximum allowed splaying angle between adjacent loudspeakers

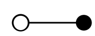


Signal Processing Model for WST



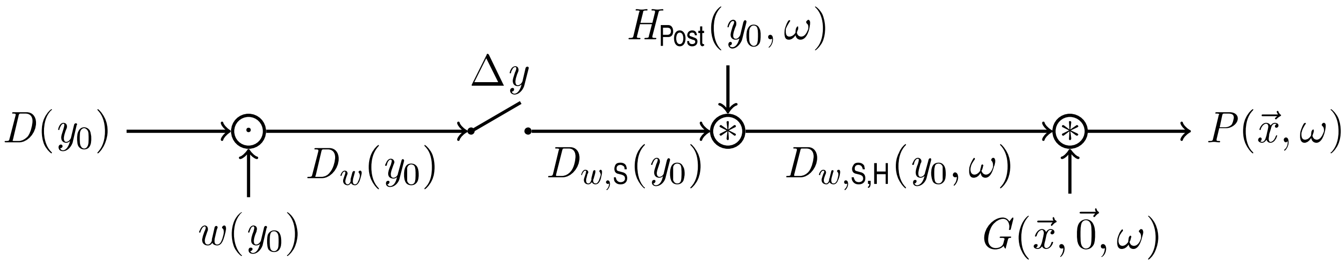
continuous, finite length line source, length L, symmetric on y-axis

$$D_w(y_0) = \begin{cases} \frac{1}{L} & \text{for } |y_0| \leq \frac{L}{2} \\ 0 & \text{else} \end{cases}$$



$$D_w(k_y, \omega) = \begin{cases} \frac{\sin(k_y \frac{L}{2})}{k_y \frac{L}{2}} & \text{for } k_y \neq 0 \\ 1 & \text{for } k_y = 0 \end{cases}$$

Signal Processing Model for WST

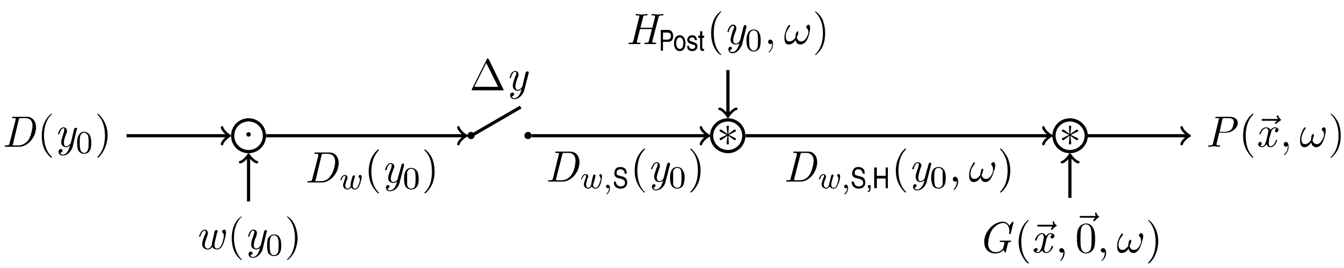


discretized, finite length line source with ideal point sources, length $L = \Delta y\, N$

$$D_{w,\text{S}}(y_0) = \sum_{\nu=-\frac{N-1}{2}}^{+\frac{N-1}{2}} \frac{1}{N} \cdot \delta(y_0 - \nu\,\Delta y) \quad \circ\!\!-\!\!\bullet$$

$$D_{w,\text{S}}(k_y,\omega) = \begin{cases} \dfrac{\sin\left(k_y\,\Delta y\,\frac{N}{2}\right)}{N\,\sin\left(k_y\,\Delta y\,\frac{1}{2}\right)} & \text{for } k_y \neq \frac{2\,\pi}{\Delta y}\,\mu \\ 1 & \text{for } k_y = \frac{2\,\pi}{\Delta y}\,\mu. \end{cases}$$

Signal Processing Model for WST

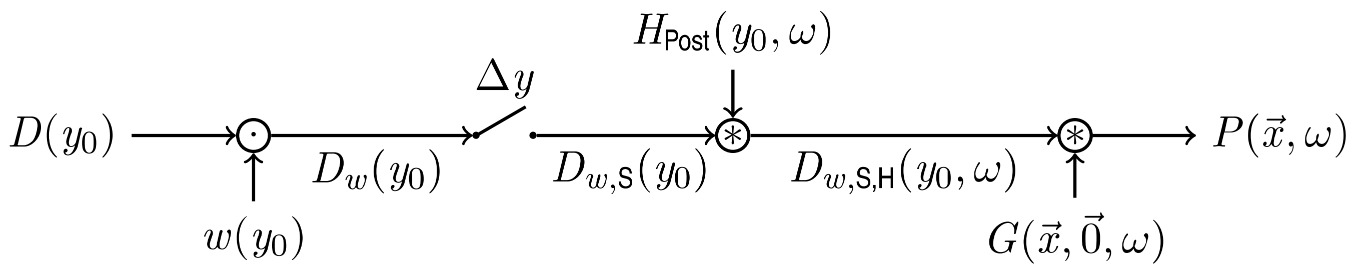


postfilter of the line piston, length l

$$H_{\text{Line}}(y_0) = \begin{cases} \frac{1}{l} & \text{for } |y_0| \leq \frac{l}{2} \\ 0 & \text{else} \end{cases} \quad \circ\!\!-\!\!\bullet$$

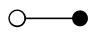
$$H_{\text{Line}}(k_y,\omega) = \begin{cases} \dfrac{\sin(k_y\,\frac{l}{2})}{k_y\,\frac{l}{2}} & \text{for } k_y \neq 0 \\ 1 & \text{for } k_y = 0, \end{cases}$$

Signal Processing Model for WST



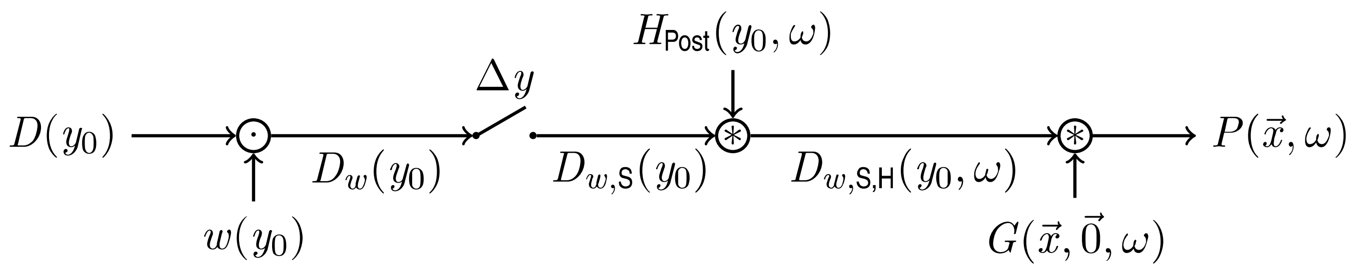
postfilter of the circular piston, radius r_0

$$H_{\text{Circ}}(y_0, z_0) = \begin{cases} \frac{1}{\pi\, r_0^2} & \text{for } \; y_0^2 + z_0^2 \leq r_0^2 \\ 0 & \text{else} \end{cases}$$



$$H_{\text{Circ}}(k_y, \omega) = \begin{cases} \frac{2\, J_1(k_y\, r_0)}{k_y\, r_0} & \text{for } \; k_y \neq 0 \\ 1 & \text{for } \; k_y = 0 \end{cases}$$

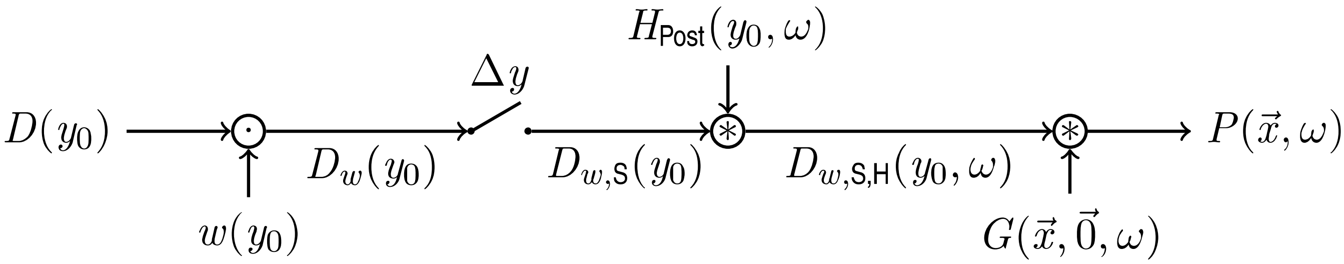
Signal Processing Model for WST



1st product theorem of array processing
spatio-temporal spectrum is obtained by a linear array using ideal point sources multiplied with the individual postfilter characteristics of pistons, when all pistons are identical:

$$D_{w,\text{S,H}}(k_y, \omega) = D_{w,\text{S}}(k_y, \omega) \cdot H_{\text{Post}}(k_y, \omega)$$

Signal Processing Model for WST

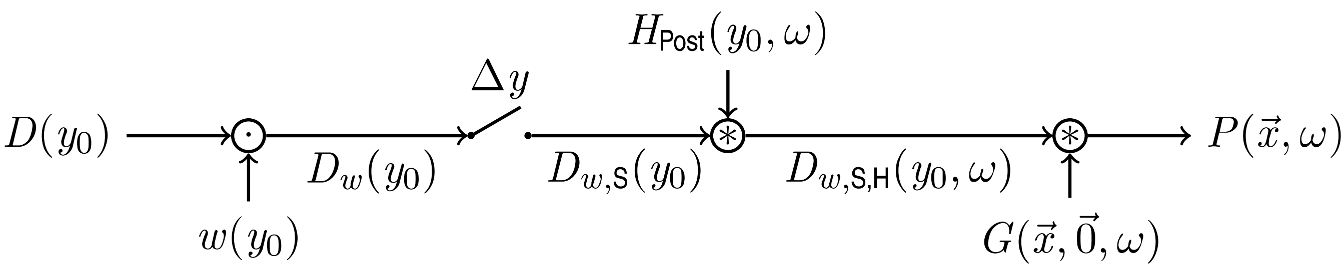


Green's function, i.e. ideal point source in the origin
for $\vec{x} = (x, y, z)^T$ and $\vec{0} = (0,0,0)^T$, here only xy-plane considered, i.e. $z = 0$

$$G(\vec{x},\vec{0},\omega) = \frac{1}{4\,\pi}\,\frac{\mathrm{e}^{-\mathrm{j}\,\frac{\omega}{c}\,\|\vec{x}-\vec{0}\|}}{\|\vec{x}-\vec{0}\|} \quad \circ\!\!-\!\!\bullet$$

$$G_0(x,k_y,z,\omega) = \begin{cases} -\frac{\mathrm{j}}{4}\,H_0^{(2)}\left(\sqrt{\left(\frac{\omega}{c}\right)^2 - k_y^2}\cdot\sqrt{x^2+z^2}\right) & \text{for } k_y^2 < \left(\frac{\omega}{c}\right)^2 \\ \frac{1}{2\,\pi}\,K_0\left(\sqrt{k_y^2 - \left(\frac{\omega}{c}\right)^2}\cdot\sqrt{x^2+z^2}\right) & \text{for } k_y^2 > \left(\frac{\omega}{c}\right)^2 \end{cases}$$

Signal Processing Model for WST



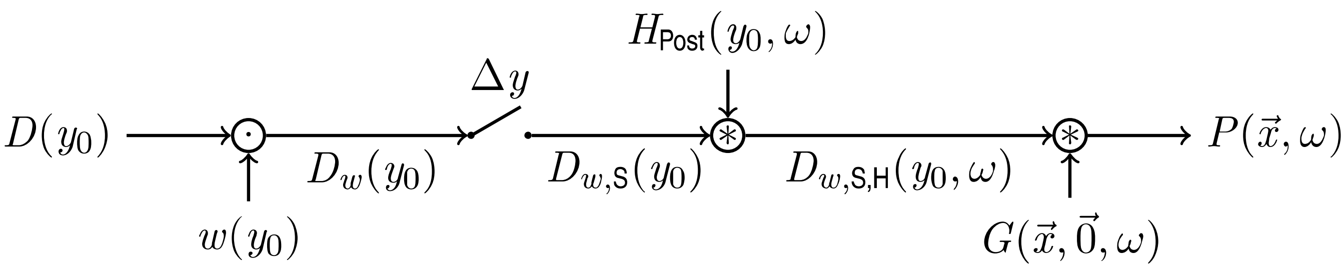
spatio-temporal spectrum of sound pressure:

$$\left\{\begin{matrix} P_w(k_y,\omega) \\ P_{w,\text{S}}(k_y,\omega) \\ P_{w,\text{S,H}}(k_y,\omega) \end{matrix}\right\} = \left\{\begin{matrix} D_w(k_y,\omega) \\ D_{w,\text{S}}(k_y,\omega) \\ D_{w,\text{S,H}}(k_y,\omega) \end{matrix}\right\} \cdot G_0(x,k_y,\omega)$$

sound field in xy-plane via inverse spatial Fourier transform:

$$\left\{\begin{matrix} P_w(x,y,\omega) \\ P_{w,\text{S}}(x,y,\omega) \\ P_{w,\text{S,H}}(x,y,\omega) \end{matrix}\right\} = \frac{1}{2\,\pi}\int_{-\infty}^{+\infty} \left\{\begin{matrix} D_w(k_y,\omega) \\ D_{w,\text{S}}(k_y,\omega) \\ D_{w,\text{S,H}}(k_y,\omega) \end{matrix}\right\} \cdot G_0(x,k_y,\omega)\,\mathrm{e}^{-\mathrm{j}\,k_y\,y}\,\mathrm{d}k_y$$

Signal Processing Model for WST

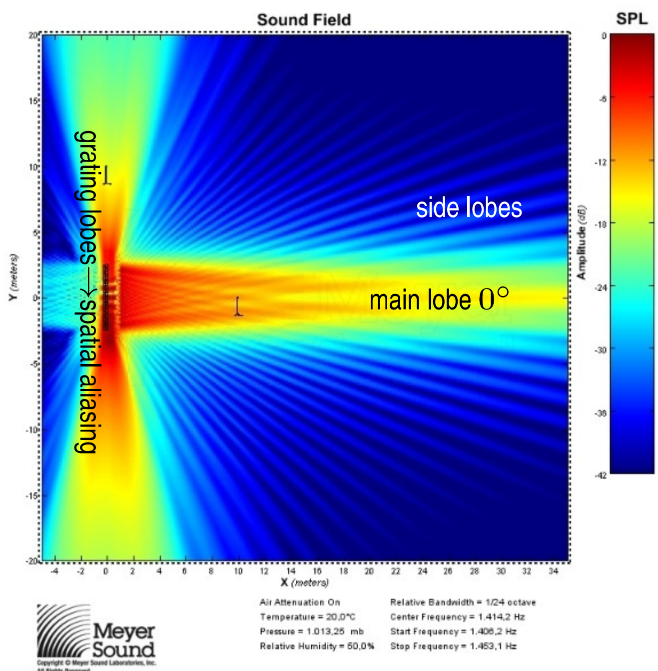


for the visible region (i.e. only propagating waves) $-\frac{\omega}{c} < k_y < +\frac{\omega}{c}$ the mapping $k_y = \frac{\omega}{c}\sin\varphi$ holds with $-90° < \varphi < +90°$

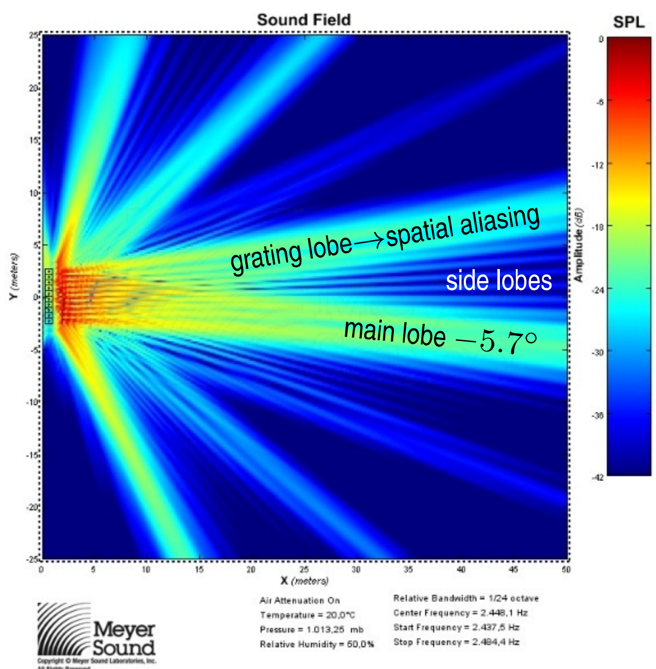
$D_w(\varphi,\omega)$, $D_{w,\mathrm{S}}(\varphi,\omega)$ and $D_{w,\mathrm{S,H}}(\varphi,\omega)$ are the interpreted as the farfield radiation patterns of the specific LSA and are proportional to the source's velocity spatio-temporal spectrum

Spatial Aliasing

Prediction with Meyer Sound MAPP Online Pro 4.4.0-9059[1]



21x M'elodie ⟶ 5 m length
no DSP, i.e. uniformly driven
f=1414 Hz, 1/24 oct.

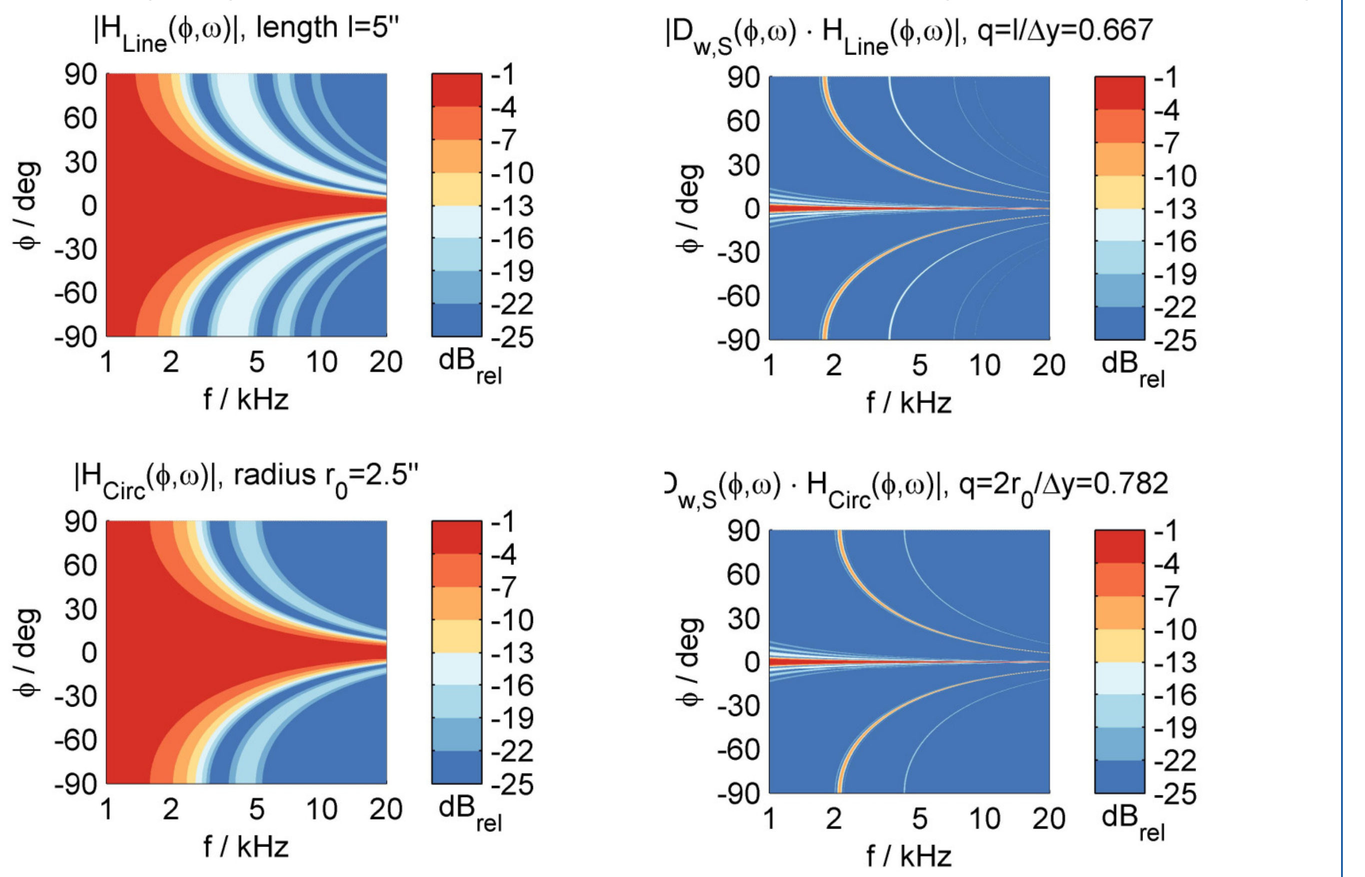


10x M3D ⟶ 5.24 m length
$\Delta\tau$=0.15 ms⟶steering φ=-5.7°
f=2448 Hz, 1/24 oct.

[1] Note that MAPP Online Pro is–in contrast to many other software–capable of revealing those artifacts. This does not imply that their products perform worse than others.

LSA with Pistons, Larger Gaps between Pistons

minimum grating lobe attenuation ≈ 7.7 dB $\rightarrow$ to be avoided, highly corrupted Fresnel region



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