Universität Rostock

Traditio et Innovatio

Sound Field Synthesis of Virtual Cylindrical Waves using Circular and Spherical Loudspeaker Arrays

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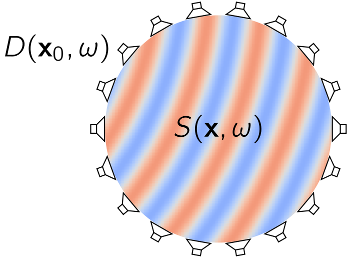
Sound Field Synthesis

aims at the physical reconstruction of a desired sound field $S(\mathbf{x}, \omega)$ within a target region using a large number of secondary sources driven by individual signals $D(\mathbf{x}_0, \omega)$



Analytic methods

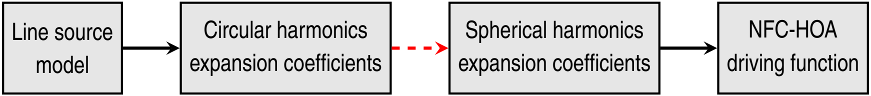
- Wave Field Synthesis (WFS)
- Near-field compensated higher-order Ambisonics (NFC-HOA)
- Spectral division method (SDM)
- ...

Analytic Source Models

- Various analytic source models are used
- Closed-form driving functions are known for

	NFC-HOA	WFS
plane wave	✓	✓
line source	✗	✓
point source	✓	✓
focused source	✓	✓
⋮		

Outline



1. spherical harmonics representation of the sound field of a line source
2. analytic NFC-HOA driving function
3. evaluation of the synthesized sound field

Circular Harmonics Representation

Line source model → Circular harmonics expansion coefficients ⇢ Spherical harmonics expansion coefficients → NFC-HOA driving function

circular harmonics expansion of a two-dimensional sound field (independent to the z-axis)

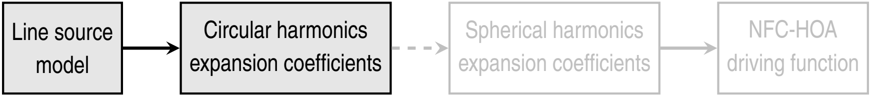
$$S(\mathbf{x},\omega) = \sum_{m=-\infty}^{\infty} \mathring{S}_m(\omega) J_m\left(\tfrac{\omega}{c} r \sin\beta\right) e^{im\alpha}$$

α: azimuth angle, β: colatitude angle

$\mathring{S}_m(\omega)$: expansion coefficient

$J_m(\cdot)$: m-th Bessel function of the first kind

Circular Harmonics Representation



circular harmonics expansion of the sound field of a line source

$$-\frac{i}{4} H_0^{(2)}\left(\frac{\omega}{c}\|\mathbf{x}-\mathbf{x}_{\mathrm{ls}}\|\right) = \sum_{m=-\infty}^{\infty} \underbrace{-\frac{i}{4} H_m^{(2)}\left(\frac{\omega}{c} r_{\mathrm{ls}}\right) e^{-im\alpha_{\mathrm{ls}}}}_{\mathring{S}_{\mathrm{ls},m}(\omega)} J_m\left(\frac{\omega}{c} r \sin\beta\right) e^{im\alpha}.$$

$\mathrm{x}_{\mathrm{ls}} = (r_{\mathrm{ls}}, \alpha_{\mathrm{ls}}, \frac{\pi}{2})$

$H_m^{(2)}(\cdot)$: m-th Hankel function of the second kind

Spherical Harmonics Representation

$$S(\mathbf{x},\omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \breve{S}_n^m(\omega) j_n(\tfrac{\omega}{c} r) Y_n^m(\beta,\alpha)$$

$\breve{S}_n^m(\omega)$: expansion coefficient

$j_n(\cdot)$: n-th spherical Bessel function of the first kind

$Y_n^m(\beta,\alpha) = \sqrt{\frac{2n+1}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\beta) e^{im\alpha}$: spherical harmonics

$P_n^m(\cdot)$: associated Legendre function

Spherical Harmonics Representation

$$S(\mathbf{x},\omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \breve{S}_n^m(\omega) j_n(\tfrac{\omega}{c} r) Y_n^m(\beta,\alpha)$$

$$= \sum_{m=-\infty}^{\infty} e^{im\alpha} \underbrace{\sum_{n=|m|}^{\infty} \breve{S}_n^m(\omega) j_n(\tfrac{\omega}{c} r) Y_n^m(\beta,0)}_{=\mathring{S}_m(\omega) J_m(\frac{\omega}{c} r \sin\beta)}$$

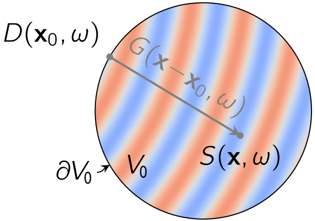
$$\boxed{\breve{S}_n^m(\omega) = 4\pi i^{m-n} Y_n^m(\tfrac{\pi}{2},0)^* \mathring{S}_m(\omega)}$$

Spherical Harmonics Representation of a Line Source

Line source model → Circular harmonics expansion coefficients → Spherical harmonics expansion coefficients → NFC-HOA driving function

$$\boxed{\breve{S}^{m}_{\mathrm{ls},n}(\omega) = -\pi i^{m-n+1} H^{(2)}_{m}\left(\tfrac{\omega}{c} r_{\mathrm{ls}}\right) Y^{m}_{n}\left(\tfrac{\pi}{2}, \alpha_{\mathrm{ls}}\right)^{*}}$$

Near-field Compensated Higher-order Ambisonics



- explicit solution of the continuous synthesis equation

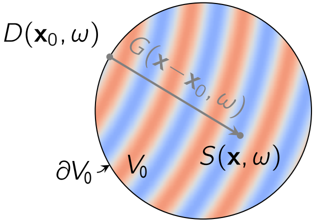
$$S(\mathbf{x}, \omega) = \oint_{\partial V_0} D(\mathbf{x}_0, \omega) G(\mathbf{x} - \mathbf{x}_0, \omega) \mathrm{d}A_0$$

- based on the spherical harmonics expansion

$$\breve{S}_n^m(\omega), \breve{G}_n^m(\omega)$$

- considers radially symmetric secondary source distribution
  - 3D: spherical distribution of point sources
  - 2D: circular distribution of line sources

Near-field Compensated Higher-order Ambisonics



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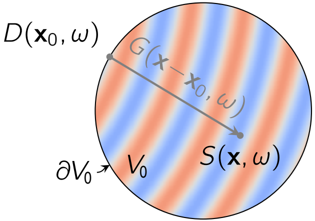
Near-field Compensated Higher-order Ambisonics



- explicit solution of the continuous synthesis equation

$$S(\mathbf{x},\omega) = \oint_{\partial V_0} D(\mathbf{x}_0,\omega) G(\mathbf{x}-\mathbf{x}_0,\omega) \mathrm{d}A_0$$

- based on the spherical harmonics expansion

$$\breve{S}_n^m(\omega), \breve{G}_n^m(\omega)$$

- considers radially symmetric secondary source distribution
  - 3D: spherical distribution of point sources
  - ~~2D: circular distribution of line sources~~
  - 2.5D: circular distribution of point sources

Driving Functions

- 3D NFC-HOA

$$D_{\text{3D}}(\alpha_0, \beta_0, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \underbrace{\frac{1}{r_0^2} \frac{\breve{S}_n^m(\omega)}{\breve{G}_n^0(\omega)}}_{\breve{D}_n^m(\omega)} Y_n^m(\beta_0, \alpha_0)$$

- 2.5D NFC-HOA

$$D_{\text{2.5D}}(\alpha_0, \omega) = \sum_{m=-\infty}^{\infty} \underbrace{\frac{1}{2\pi r_0} \frac{\breve{S}_{|m|}^m(\omega)}{\breve{G}_{|m|}^m(\omega)}}_{\mathring{D}_m(\omega)} e^{im\alpha_0}$$

Driving Functions

- 3D NFC-HOA (2D sound field)

$$D_{3D}(\alpha_0, \beta_0, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \underbrace{\frac{1}{r_0^2} \frac{4\pi i^{m-n} Y_n^m(\frac{\pi}{2}, 0)^* \mathring{S}_m(\omega)}{\breve{G}_n^0(\omega)}}_{\breve{D}_n^m(\omega)} Y_n^m(\beta_0, \alpha_0)$$

- 2.5D NFC-HOA (2D sound field)

$$D_{2.5D}(\alpha_0, \omega) = \sum_{m=-\infty}^{\infty} \underbrace{\frac{1}{2\pi r_0} \frac{4\pi i^{m-|m|} Y_{|m|}^m(\frac{\pi}{2}, 0)^* \mathring{S}_m(\omega)}{\breve{G}_{|m|}^m(\omega)}}_{\mathring{D}_m(\omega)} e^{im\alpha_0}$$

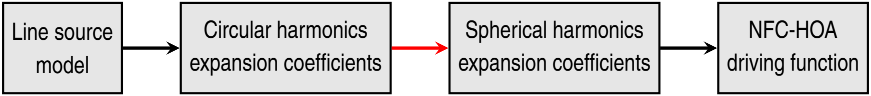
Driving Functions

- 3D NFC-HOA (line source)

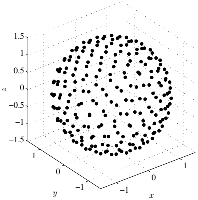
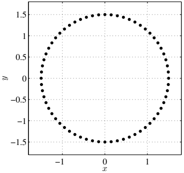
$$D_{\mathrm{3D}}(\alpha_0, \beta_0, \omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \underbrace{\frac{1}{r_0^2} \frac{-\pi i^{m-n+1} H_m^{(2)}(\frac{\omega}{c} r_{\mathrm{ls}}) Y_n^m(\frac{\pi}{2}, \alpha_{\mathrm{ls}})^*}{\breve{G}_n^0(\omega)}}_{\breve{D}_n^m(\omega)} Y_n^m(\beta_0, \alpha_0)$$

- 2.5D NFC-HOA (line source)

$$D_{\mathrm{2.5D}}(\alpha_0, \omega) = \sum_{m=-\infty}^{\infty} \underbrace{\frac{1}{2\pi r_0} \frac{-\pi i^{m-|m|+1} H_m^{(2)}(\frac{\omega}{c} r_{\mathrm{ls}}) Y_{|m|}^m(\frac{\pi}{2}, \alpha_{\mathrm{ls}})^*}{\breve{G}_{|m|}^m(\omega)}}_{\mathring{D}_m(\omega)} e^{im\alpha_0}$$



Numerical Simulation

	3D NFC-HOA	2.5D NFC-HOA
r_0	1.5 m	1.5 m
$N_{\text{loudspeaker}}$	484[1]	64
maximum order	21	31
$f_{\text{artifact-free}}$	764 Hz	1128 Hz
		

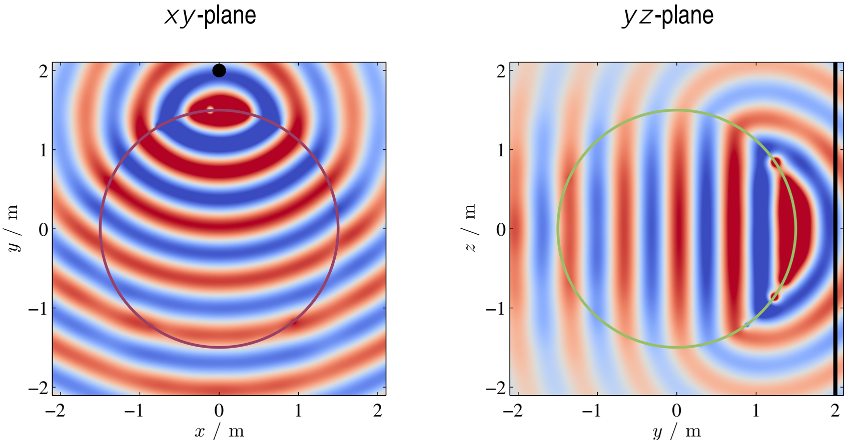
Sound Field Synthesis toolbox (`https://github.com/sfstoolbox/sfs`)

secondary monopole sources $\breve{G}_n^m(\omega) = -i\frac{\omega}{c} h_n^{(2)}(\frac{\omega}{c} r) Y_n^m(\beta_0, \alpha_0)^*$

[1] Riesz s-energy approach

3D Synthesis

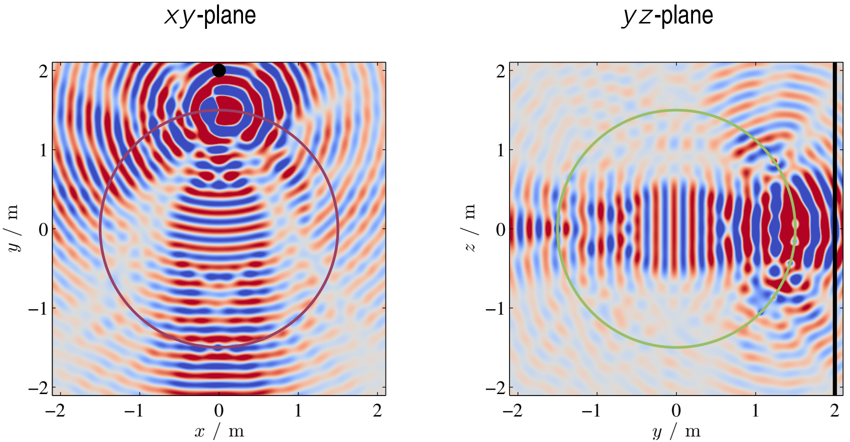
500 Hz



$r_0 = 1.5$ m, $N_{\text{loudspeaker}} = 484$, $M = 21$, $\mathrm{x}_{\text{ls}} = (r_{\text{ls}}, \alpha_{\text{ls}}, \frac{\pi}{2})$

3D Synthesis

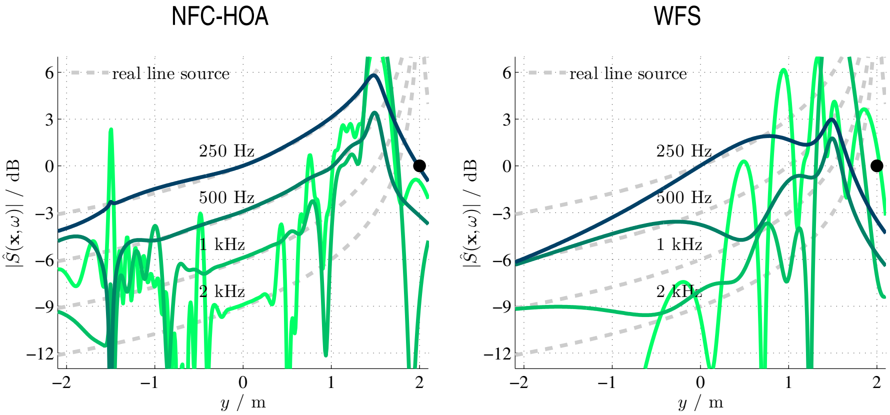
1500 Hz



$r_0 = 1.5$ m, $N_{\text{loudspeaker}} = 484$, $M = 21$, $\mathrm{x}_{\text{ls}} = (r_{\text{ls}}, \alpha_{\text{ls}}, \frac{\pi}{2})$

3D Synthesis

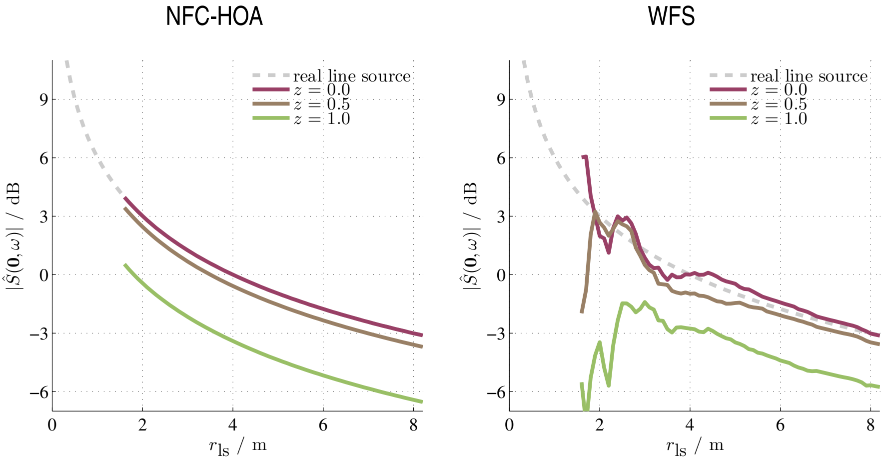
Amplitude Decay for Fixed Source Position



$r_0 = 1.5$ m, $N_{\text{loudspeaker}} = 484$, $M = 21$, $\mathrm{x}_{\text{ls}} = (r_{\text{ls}}, \alpha_{\text{ls}}, \frac{\pi}{2})$

3D Synthesis

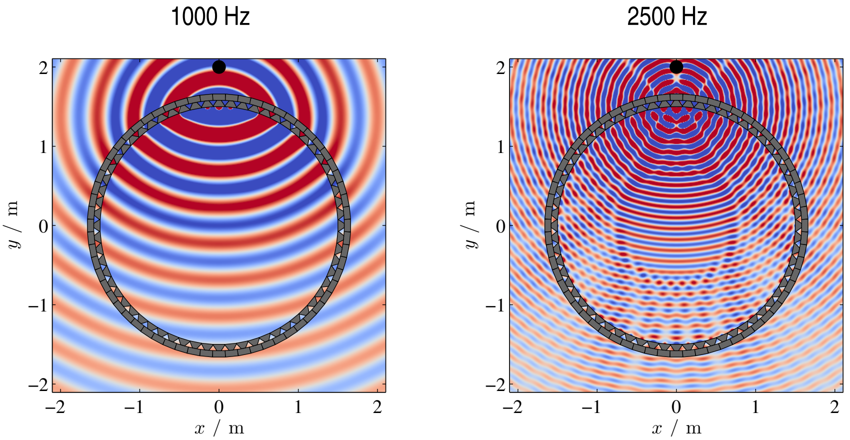
Amplitude Decay for Varying Source Position



$r_0 = 1.5$ m, $N_{\mathrm{loudspeaker}} = 484$, $M = 21$, $\mathrm{x}_{\mathrm{ls}} = (r_{\mathrm{ls}}, \alpha_{\mathrm{ls}}, \frac{\pi}{2})$ $f = 500$ Hz

2.5D Synthesis

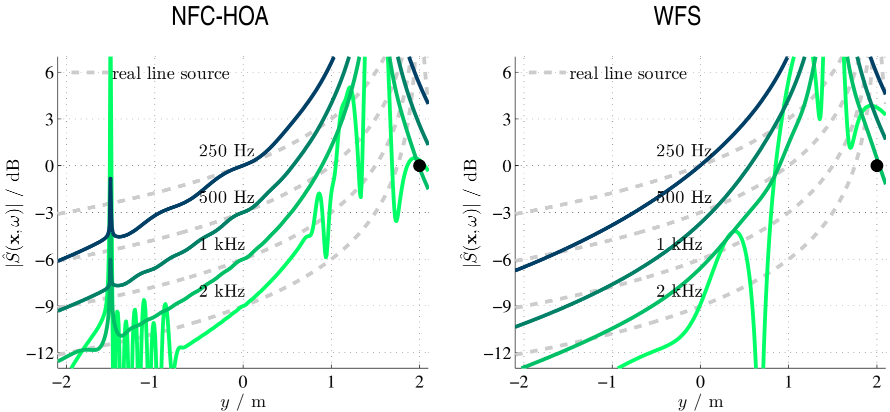
NFC-HOA



$r_0 = 1.5$ m, $N_{\text{loudspeaker}} = 64$, $M = 31$, $\mathrm{x}_{\text{ls}} = (r_{\text{ls}}, \alpha_{\text{ls}}, \frac{\pi}{2})$

2.5D Synthesis

Amplitude Decay



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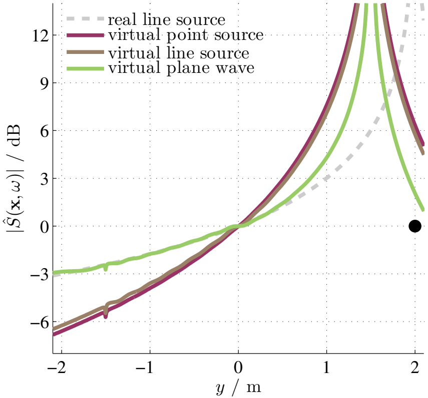
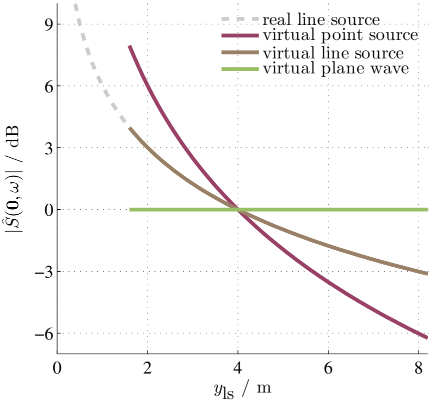
2.5D Synthesis

Amplitude Decay

Fixed source position

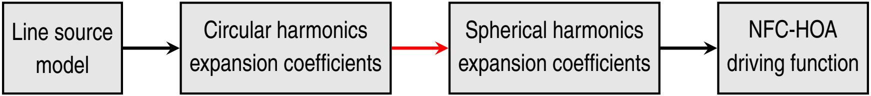


Varying source position



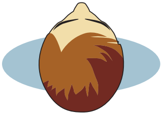
$r_0 = 1.5$ m, $N_{\mathrm{loudspeaker}} = 64$, $M = 31$, $\mathrm{x}_{\mathrm{ls}} = (r_{\mathrm{ls}}, \alpha_{\mathrm{ls}}, \frac{\pi}{2})$

Summary and Discussion



- sound field with a mild amplitude decay
- compensation of the low-pass characteristic ($F_{EQ}(\omega) = \sqrt{i\frac{\omega}{c}}$)
- efficient realization of driving function required

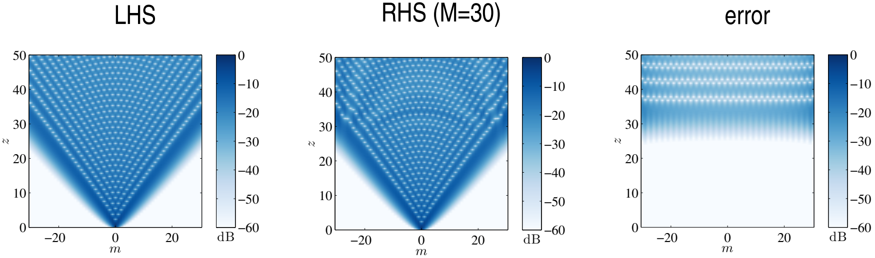
THANK YOU!



http://spatialaudio.net

A1. Converting $\mathring{S}_m(\omega)$ to $\breve{S}_n^m(\omega)$

$$i^{-m} J_m(z) = \sum_{n=|m|}^{\infty} 4\pi i^{-n} j_n(z) Y_n^m(\tfrac{\pi}{2}, 0)^* Y_n^m(\beta, 0)$$



A2. Green's Function

Spherical harmonics expansion of $G(\mathbf{x} - \mathbf{x}_0, \omega)$

- 3D NFC-HOA: $\mathbf{x}_0 = (r_0, 0, 0)$
- 2.5D NFC-HOA: $\mathbf{x}_0 = (r_0, 0, \frac{\pi}{2})$

Free-field Green's function

$$\breve{G}_n^m(\omega) = -i\frac{\omega}{c} h_n^{(2)}(\frac{\omega}{c} r_0) Y_n^m(\beta_0, \alpha_0)^*$$