



On the Connections of Wave Field Synthesis and Spectral Division Method Plane Wave Driving Functions

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42. DAGA, Aachen

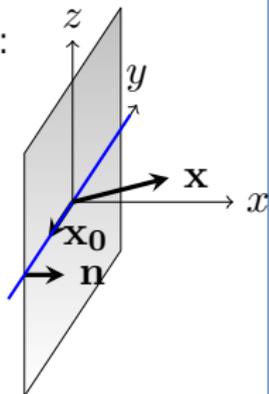
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Introduction

Spectral Division Method (SDM) is the **explicit** solution / **inverse** wavefield propagator
Wave Field Synthesis (WFS) is the **implicit** solution / **forward** wavefield propagator

Linear array on y -axis \rightarrow 2.5D Sound Field Synthesis (SFS) in xy -plane:

$$P(x, y, \omega) = \int_{-\infty}^{+\infty} D(y_0, \omega) \frac{e^{-j \frac{\omega}{c} \sqrt{x^2 + (y - y_0)^2}}}{4 \pi \sqrt{x^2 + (y - y_0)^2}} dy_0$$



Virtual Point Source

Farfield / high-frequency approximated 2.5D SDM [Spors PreEQ AES 2010] \equiv

Neumann WFS 3D \rightarrow 2.5D with Stationary Phase Approximations (SPAs) I+II [Start PhD 1997]

Virtual Plane Wave

mismatch reported in literature [Ahrens SDM IEEE 2010]

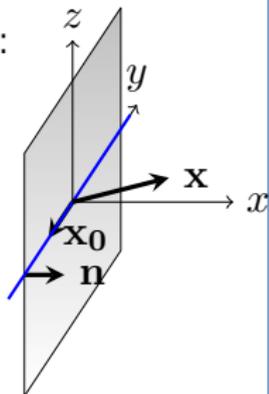
$$D_{\text{WFS}}(y_0, \omega) = D_{\text{SDM, HF/Far}}(y_0, \omega) \sqrt{\cos \varphi_{\text{PW}}}$$

Introduction

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Neumann WFS 3D \rightarrow 2.5D with Stationary Phase Approximations (SPAs) I+II [Start PhD 1997]

Virtual Plane Wave

no mismatch

Farfield / high-frequency approximated 2.5D SDM [Ahrens SDM IEEE 2010] \equiv

Neumann WFS 3D \rightarrow 2.5D with Stationary Phase Approximations (SPAs) I+II [Schultz PhD 2016]

3D SFS with a Planar Array, SDM vs. WFS

Spectral Division Method (SDM):

$$P(x, y, z, \omega) = \iint_{-\infty}^{+\infty} D(y_0, z_0, \omega) \frac{e^{-j \frac{\omega}{c} \sqrt{x^2 + (y-y_0)^2 + (z-z_0)^2}}}{4 \pi \sqrt{x^2 + (y-y_0)^2 + (z-z_0)^2}} dz_0 dy_0$$

$$D(k_y, k_z, \omega) = \frac{P(x, k_y, k_z, \omega)}{G_0(x, k_y, k_z, \omega)}$$

Wave Field Synthesis with Neumann Rayleigh integral (Neumann WFS):

$$P(x, y, z, \omega) = \iint_{-\infty}^{+\infty} -\frac{\partial S(\mathbf{x}_0)}{\partial n} \frac{2 \cdot e^{-j \frac{\omega}{c} \sqrt{x^2 + (y-y_0)^2 + (z-z_0)^2}}}{4 \pi \sqrt{x^2 + (y-y_0)^2 + (z-z_0)^2}} dz_0 dy_0$$

SDM \equiv Neumann WFS cf. [Lalor 1968]

2.5D SFS with a Linear Array, SDM vs. WFS

Spectral Division Method (SDM):

$$P(x, y, \omega) = \int_{-\infty}^{+\infty} D(y_0, \omega) \frac{e^{-j \frac{\omega}{c} \sqrt{x^2 + (y-y_0)^2}}}{4 \pi \sqrt{x^2 + (y-y_0)^2}} dy_0$$

$$D(x_{\text{Ref}}, k_y, \omega) = \frac{P(x_{\text{Ref}}, k_y, \omega)}{G_0(x_{\text{Ref}}, k_y, \omega)}$$

Wave Field Synthesis with Neumann Rayleigh integral (Neumann WFS):

$$P(x, y, \omega) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} -\frac{\partial S(\mathbf{x}_0)}{\partial n} \frac{2 \cdot e^{-j \frac{\omega}{c} \sqrt{x^2 + (y-y_0)^2 + (z-z_0)^2}}}{4 \pi \sqrt{x^2 + (y-y_0)^2 + (z-z_0)^2}} dz_0 \right] dy_0$$

SDM vs. Neumann WFS ???

[Start PhD 1997, Ahrens SDM IEEE 2010, Spors PreEQ AES 2010, Völk PSC AES 2012, Firtha JAES 2015]

Stationary Phase Approximation I : Reference Point

$$P(\mathbf{x}, \omega) = \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} -2 \frac{\partial P(\mathbf{x}'_0, \omega)}{\partial n} G_{0,3D}(\mathbf{x}, \mathbf{x}'_0, \omega) dz_0 \right] dy_0$$

with $\mathbf{x}'_0 = (0, y_0, z_0)^T$, $\mathbf{x} = (x, y, 0)^T$, $\mathbf{n} = (1, 0, 0)^T$

[Sta97, (3.2)], [Wil99, (4.93)]:

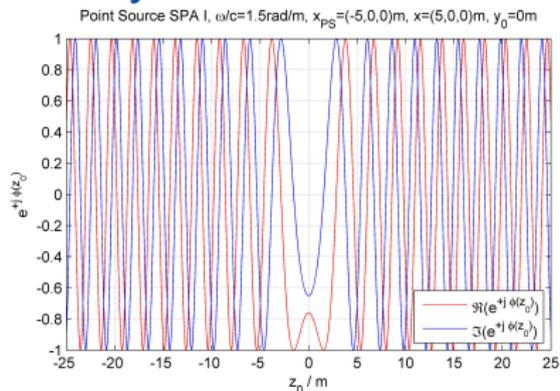
$$I = \int_{-\infty}^{+\infty} f(z_0) e^{+j\phi(z_0)} dz_0$$

$$\phi'(z_0 = z_{0,s}) = 0 \quad \phi''(z_0 = z_{0,s}) \neq 0$$

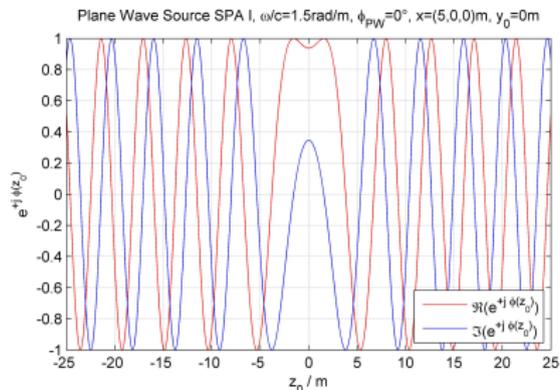
[Sta97, (3.3)], [Wil99, (4.97)]:

$$I \approx \sqrt{\frac{2\pi}{|\phi''(z_{0,s})|}} f(z_{0,s}) e^{+j\phi(z_{0,s})} e^{+j\frac{\pi}{4} \cdot \text{sign}[\phi''(z_{0,s})]}$$

Example of Stationary Phase Point for SPA I



stationary phase point $z_{0,s} = 0$



Driving Functions for Reference Point

Virtual point source with SPA I [Sta97, (3.10&3.11)]

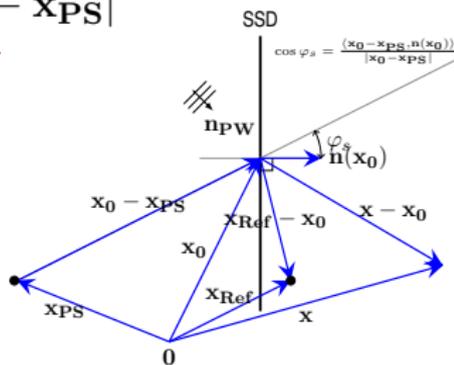
$$D(\mathbf{x}_0, \omega) = P(\omega) \sqrt{\frac{j \frac{\omega}{c}}{2\pi}} e^{-j \frac{\omega}{c} |\mathbf{x}_0 - \mathbf{x}_{PS}|} \frac{\langle \mathbf{x}_0 - \mathbf{x}_{PS}, \mathbf{n}(\mathbf{x}_0) \rangle}{\sqrt{|\mathbf{x}_0 - \mathbf{x}_{PS}|} |\mathbf{x}_0 - \mathbf{x}_{PS}|} \times$$

$$\sqrt{\frac{|\mathbf{x}_{Ref} - \mathbf{x}_0|}{|\mathbf{x}_0 - \mathbf{x}_{PS}| + |\mathbf{x}_{Ref} - \mathbf{x}_0|}}$$

Virtual plane wave with SPA I [Spo06, (5.4&5.5)], [Spo08, (27)]

$$D(\mathbf{x}_0, \omega) = P(\omega) \sqrt{8\pi j \frac{\omega}{c}} e^{-j \frac{\omega}{c} \langle \mathbf{npw}, \mathbf{x}_0 \rangle} \times$$

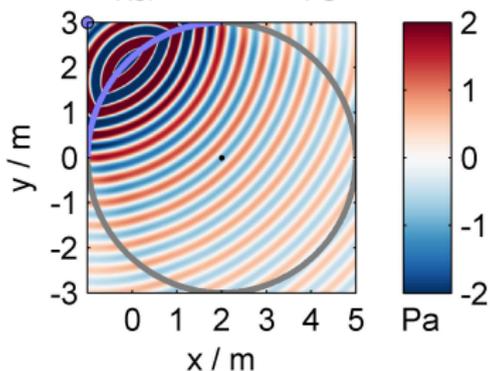
$$\sqrt{|\mathbf{x}_{Ref} - \mathbf{x}_0|} \langle \mathbf{npw}, \mathbf{n}(\mathbf{x}_0) \rangle$$



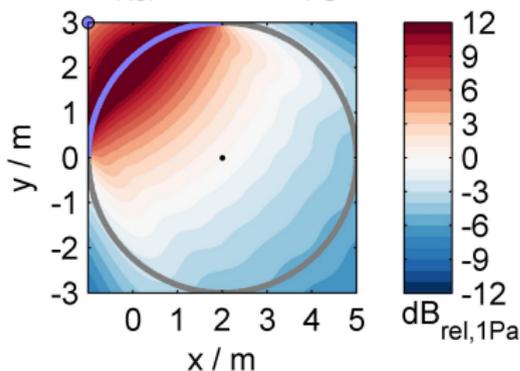
coplanarity of $\mathbf{x} - \mathbf{x}_0$, $\mathbf{x}_{Ref} - \mathbf{x}_0$, $\mathbf{n}(\mathbf{x}_0)$ and $\mathbf{x}_0 - \mathbf{x}_{PS}$ resp. \mathbf{npw}

2.5D SFS with SPA I: Reference Point

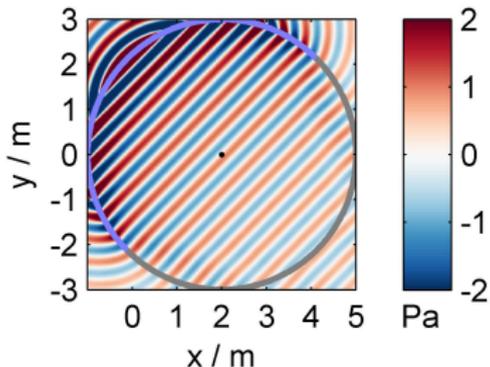
RefPnt@ $x_{\text{Ref}}=(2,0,0)^T, x_{\text{PS}}=(-1,3,0)^T$



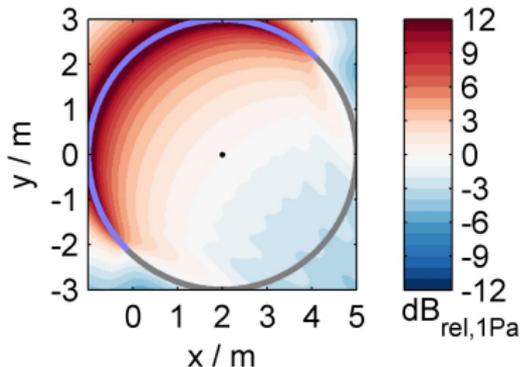
RefPnt@ $x_{\text{Ref}}=(2,0,0)^T, x_{\text{PS}}=(-1,3,0)^T$



RefPoint@ $x_{\text{Ref}}=(2,0,0)^T, \phi_{\text{PW}}=-45^\circ$

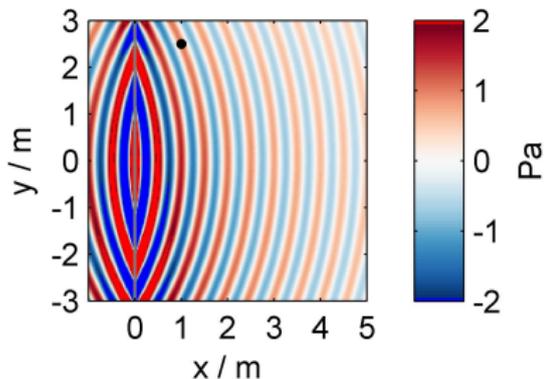


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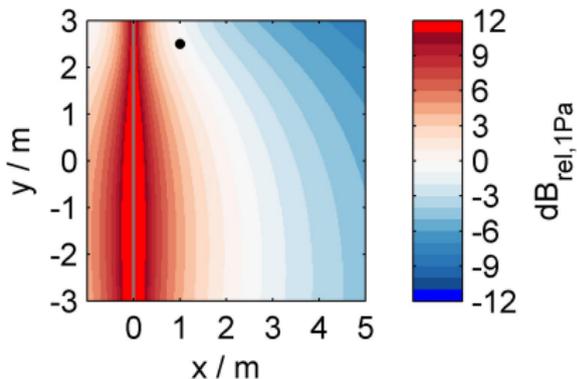


2.5D SFS with SPA I: Reference Point

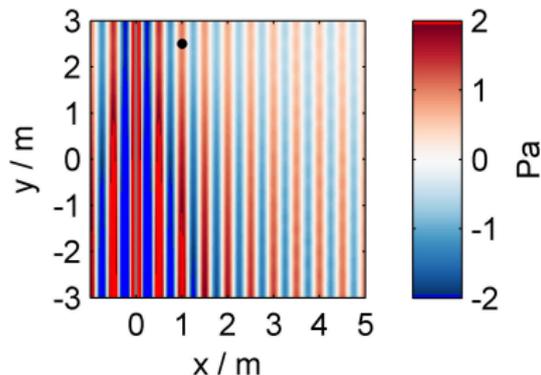
RefPoint, $x_{PS}=(-5,0)$, $x_{Ref}=(1,2.5)$



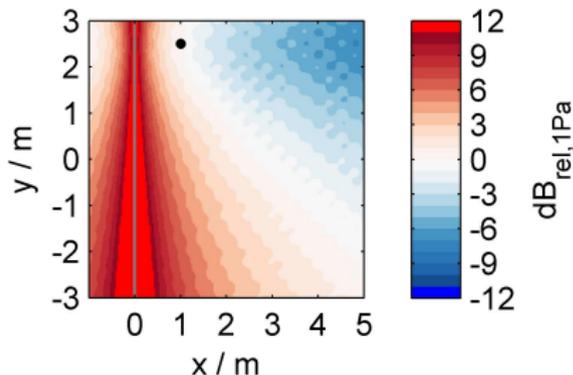
RefPoint, $x_{PS}=(-5,0)$, $x_{Ref}=(1,2.5)$



RefPoint, $\phi_{PW}=0^\circ$, $x_{Ref}=(1,2.5)$



RefPoint, $\phi_{PW}=0^\circ$, $x_{Ref}=(1,2.5)$



Stationary Phase Approximation II : Reference Line

$$P(\mathbf{x}, \omega) = \int_{-\infty}^{+\infty} D_{\text{RefPoint}}(\mathbf{x}_0, \omega) G_{0,3D}(\mathbf{x}, \mathbf{x}_0, \omega) dy_0$$

with $\mathbf{x}_0 = (0, y_0, 0)^T$, $\mathbf{x} = (x, y, 0)^T$, $\mathbf{n} = (1, 0, 0)^T$

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$$g_{\text{SPA I}} \xrightarrow{y_{0,s}} g_{\text{SPA II}}$$

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$g_{\text{SPA I}} \xrightarrow{y_{0,s}} g_{\text{SPA II}}$

for plane wave:

$$(y - y_{0,s})^2 = x^2 \tan^2 \varphi_{\text{PW}}$$

$$|\mathbf{x} - \mathbf{x}_0| = x \sqrt{1 + \tan^2 \varphi_{\text{PW}}}$$

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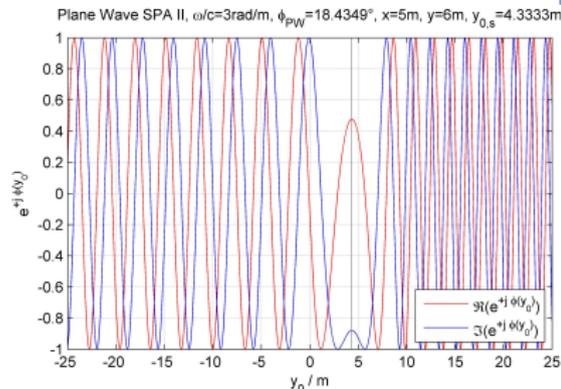
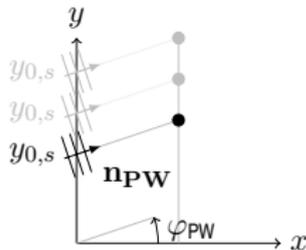
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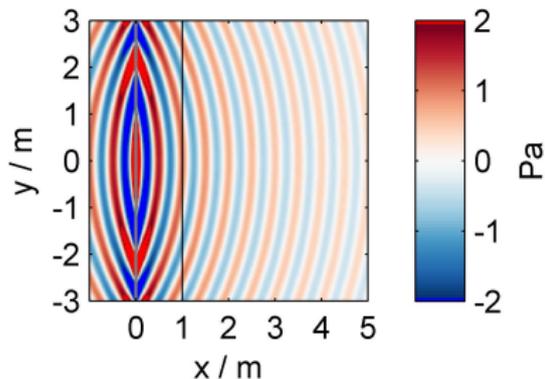
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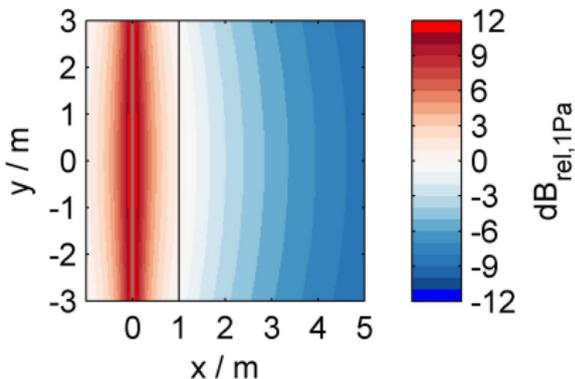


2.5D SFS with SPA II: Reference Line

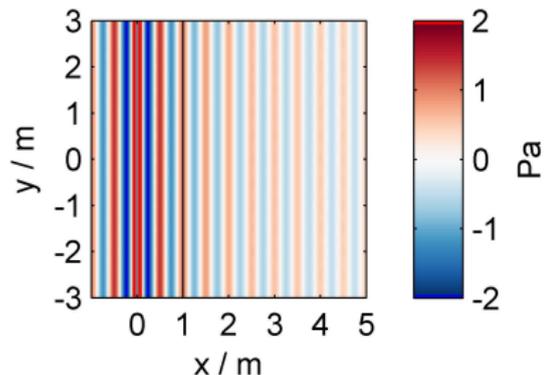
RefLine, $x_{PS}=(-5,0)$, $x_{Ref}=1$



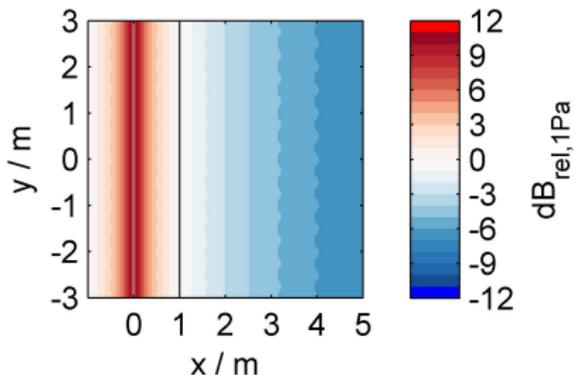
RefLine, $x_{PS}=(-5,0)$, $x_{Ref}=1$



RefLine, $\phi_{PW}=0^\circ$, $x_{Ref}=1$



RefLine, $\phi_{PW}=0^\circ$, $x_{Ref}=1$



Mismatch WFS vs. SDM for the Plane Wave ??

Parallel reference line: SPA II, WFS 2x Far/HF [Sch16, (2.183)] \equiv SDM 1x Far/HF [Ahr10, (29)]

$$D(\mathbf{x}_0, r_{\text{Ref}}, \omega) = P(\omega) \sqrt{8\pi j \frac{\omega}{c}} e^{-j \frac{\omega}{c} \langle \mathbf{n}_{\text{PW}}, \mathbf{x}_0 \rangle} \sqrt{r_{\text{Ref}}} \sqrt{\langle \mathbf{n}_{\text{PW}}, \mathbf{n}(\mathbf{x}_0) \rangle}$$

Reference point: SPA I, WFS 1x Far/HF [Spo08, (27)]

$$D(\mathbf{x}_0, \mathbf{x}_{\text{Ref}}, \omega) = P(\omega) \sqrt{8\pi j \frac{\omega}{c}} e^{-j \frac{\omega}{c} \langle \mathbf{n}_{\text{PW}}, \mathbf{x}_0 \rangle} \sqrt{|\mathbf{x}_{\text{Ref}} - \mathbf{x}_0|} \langle \mathbf{n}_{\text{PW}}, \mathbf{n}(\mathbf{x}_0) \rangle$$

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setting $|\mathbf{x}_{\text{Ref}} - \mathbf{x}_0| = r_{\text{Ref}}$ is an **invalid stationary phase point** and produces a **mismatch**

$$\text{e.g. } |\mathbf{x}_{\text{Ref}} - \mathbf{x}_0| = \left| \begin{pmatrix} x \\ y_0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ y_0 \\ 0 \end{pmatrix} \right| = x$$

$$\sqrt{\langle \mathbf{n}_{\text{PW}}, \mathbf{n}(\mathbf{x}_0) \rangle} \text{ vs. } \langle \mathbf{n}_{\text{PW}}, \mathbf{n}(\mathbf{x}_0) \rangle$$

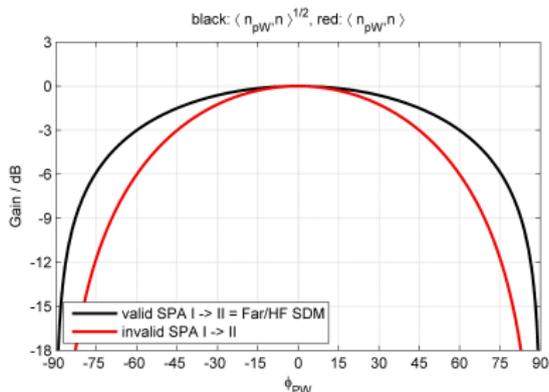
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Conclusion

- 2.5D SFS with driving functions either for **reference point** or for **reference line**
- Neumann WFS with SPA I \equiv reference point, most suitable for circular SSDs
- Neumann WFS with SPA I+II \equiv SDM Far/HF \equiv parallel reference line
- SDM \equiv WFS also holds for moving point source [Firtha JAES 2015]
- no mismatch for plane wave WFS vs. SDM when considering the correct stationary phase point along SSD
- SDM is the exact solution, inverse spatial FT mostly not known
- linear array, reference line practical?

slides @ <http://spatialaudio.net>



References

[Ahr10, Lal68, Rab06, Sch16, Spo06, Spo08, Spo10, Sta97, Völ12, Wil99, Fir15a, Fir15b]

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